A Topological Representation Theorem for Tropical Oriented Matroids

FPSAC 2012, Nagoya

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2 August 2012
Oriented Matroids

- arrangements of real hyperplanes
- covectors: describe position relative to the hyperplanes
- oriented matroid (OM): combinatorial model for the set of covectors
- non-realisable OMs

Theorem („Topological Representation Theorem“, Folkman & Lawrence, 1978)
Every OM can be realised as an arrangement of pseudohyperplanes.
We want a similar theory in the tropical world!

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- named “tropical” in honour of Brazilian mathematician Imre Simon
- algebraic geometry over the tropical semiring \((\mathbb{R} \cup \{\infty\}, \oplus, \odot)\)
  \[ x \oplus y := \min\{x, y\}, \quad x \odot y := x + y \]
- linear tropical polynomial: \(p(x) = \bigoplus_{i=1}^{d} a_i \odot x_i = \min_{1 \leq i \leq d} \{a_i + x_i\}\)
- vanishing locus / tropical hypersurface: minimum attained twice
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(1,3,2)
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Tropical Oriented Matroids (TOMs) and Tropical Pseudohyperplanes

- Definition by Ardila and Develin via covector-axioms
  - Ardila, Develin: The types in an arrangement of tropical hyperplanes yield a TOM.
  - There are non-realisable TOMs.
  - Analogue to the Topological Representation Theorem?

**Definition**

A tropical pseudohyperplane (TROPHY) is the image of a tropical hyperplane under a PL homeomorphism of $\mathbb{T}P^{d-1}$ that fixes the boundary.

**Problem:** Define tropical pseudohyperplane arrangements!
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**Definition**
A polytopal subdivision of $n\triangle^{d-1}$ is mixed if every face is a Minkowski sum of faces of $\triangle^{d-1}$.

**Theorem (Ardila, Develin, 2007)**
Every TOM yields a mixed subdivision.

**Conjecture (Ardila, Develin, 2007)**
The converse also holds.
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tropical hyperplane arrangements

mixed subdivisions of $n\Delta^{d-1}$

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A/D: 3 out of 4 axioms
Oh/Yoo: fine case

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“The Bigger Picture”
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Topological Representation Theorem ???

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Mixed Subdivisions and TROPHYs

Theorem (H., 2011)

The Poincaré dual of a mixed subdivision of $n\triangle^{d-1}$ yields a family of tropical pseudohyperplanes.
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Let $M$ be a TOM.

**Elimination property**: For $A, B \in M$, $k \in [n]$ there is $C \in M$ such that

- $C_k = A_k \cup B_k$,
- $C_i \in \{A_i, B_i, A_i \cup B_i\}$.

**convex hull of $A$ and $B$**: $M_{AB} := \{ C \in M \mid C_i \in \{A_i, B_i, A_i \cup B_i\}\}$.

Contains every elimination of $A$ and $B$.

**Theorem (H., 2010)**

A mixed subdivision $S$ has the elimination property $\iff S_{AB}$ is path-connected for all $A, B \in S$.

$\Rightarrow$ This is a topological problem!
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Arrangements of Tropical Pseudohyperplanes (TROPHYs)

**IDEA:** Represent convex hull as intersection of affine pseudohalfspaces.

Definition (H., 2010/2011)
A finite family $\mathcal{A}$ of TROPHYs is an arrangement if for every $\mathcal{A}' \subseteq \mathcal{A}$ and $\mathcal{I}$

- $\bigcap \mathcal{A}'_{\mathcal{I}}$ is empty or
- $\mathcal{A}'_{\mathcal{I}}$ is an arrangement of linear pseudohyperplanes.

Theorem (Topological Representation Theorem, H., 2011)
A mixed subdivision of $n \triangle^{d-1}$ yields an arrangement of TROPHYs.

$\mathcal{I} = (2, 23, 1, 12, 3, 3)$
$\mathcal{A}_{\mathcal{I}}$: induced family of (linear) pseudohyperplanes
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"The Bigger Picture" revisited

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Ardila/Develin

Topological Representation Theorem
“The Bigger Picture” revisited

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Ardila/Develin

Topological Representation Theorem

2 August 2012 | TU Darmstadt | Silke Horn | 10
“The Bigger Picture” revisited

- Tropical oriented matroids
- Mixed subdivisions of $n \triangle^{d-1}$
- Tropical hyperplane arrangements
- Tropical pseudohyperplane arrangements

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Topological Representation Theorem !!!
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Corollary !!!

Topological Representation Theorem !!!
The Missing Arrow
The Elimination Property

Theorem (H., 2011)

*Tropical pseudohyperplane arrangements satisfy the elimination property.*

Sketch of proof.

- **convex hull of types:**

  \[ \text{conv}(A, B) := \{ C \mid C_i \in \{ A_i, B_i, A_i \cup B_i \} \} \]

- Elimination is satisfied iff convex hull is path-connected.

- Approximate \( \text{conv}(A, B) \) by affine pseudohalfspaces.

- Constructed by “blowing up” tropical pseudohyperplanes.

- Apply Topological Representation Theorem.
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Thanks for your attention!