Development of Moduli Theory: Lecture Series

June 11 – 14, 2013

Kyoto University, RIMS, Room 420

(Last modified: May 31, 2013)

○ **Igor Dolgachev (University of Michigan)**
Introduction to the theory of Enriques sufraces

○ **Eduard Looijenga (Utrecht University)**
Moduli spaces and Shimura varieties

○ **Hiraku Nakajima (Kyoto University)**
Instanton and representation theory

○ **Iku Nakamura (Hokkaido University)**
Compactification of the moduli of abelian varieties

○ **Carlos Simpson (Université de Nice-Sophia Antipolis)**
A study of moduli of stable bundles

Program

	June 11	**June 12**	**June 13**	**June 14**
10:00 – 11:00	Nakamura	Nakamura	Nakamura	Looijenga
11:30 – 12:30	Dolgachev	Dolgachev	Dolgachev	Nakajima
14:30 – 15:30	Looijenga	Simpson	Nakajima	Simpson
16:00 – 17:00	Nakajima	Looijenga	Simpson	————

Organizer:

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Abstract

Igor Dolgachev (University of Michigan)

Introduction to the theory of Enriques sufraces

After reminding some generalities about Enriques surfaces over an algebraically closed field of arbitrary characteristic (Lecture 1) we will go more in depth discussing nodal invariants that give information about smooth rational curves on an Enriques surface (Lecture 2) and also the theory of moduli of polarized and unpolarized Enriques surfaces (Lecture 3).

Eduard Looijenga (Utrecht University)

Moduli spaces and Shimura varieties

Hiraku Nakajima (Kyoto University)

Instanton and representation theory

Iku Nakamura (Hokkaido University)

Compactification of the moduli of abelian varieties

Let $\mathbf{P}_k^2$ be the projective plane over an algebraically closed field k of characteristic different from 3. A Hesse cubic curve is a cubic curve on the plane $\mathbf{P}_k^2$ defined by the equation:

$$C(\mu) \ : \ x_0^3 + x_1^3 + x_2^3 - 3\mu x_0 x_1 x_2 = 0$$

for some $\mu \in \mathbf{P}^1(k)$. The set of isomorphism classes of all Hesse cubics is identified as $\mathbf{P}^1$. This is one of the simplest examples of the complete moduli spaces of abelian varieties. A remarkable fact is that any of Hesse cubic curves is GIT-stable in the sense that its $PGL(3)$-orbit is closed in the space of all semistable cubics. The purpose of this lecture is first to generalize this to arbitrary dimension to construct a sort of a complete fine moduli scheme of abelian schemes with noncommutaive level structures. We explain also how they are related to classical theta functions. Second, we construct another canonical complete coarse moduli scheme of abelian varieties with similar level structures. We also compare these two moduli spaces in a functorial way. The references for the topic are

[1] I. Nakamura, Planar cubic curves, from Hesse to Mumford, Sugaku Expositions **17** (2004), 73–101.

[2] I. Nakamura, Stability of degenerate abelian varieties, Invent. Math. **136** (1999), 659–715.

Please visit our homepage:

`http://www.math.sci.hokudai.ac.jp/~nakamura/`
`http://www.math.sci.hokudai.ac.jp/~nakamura/amsp3020703.pdf`

Carlos Simpson (Université de Nice-Sophia Antipolis)

A study of moduli of stable bundles

We will consider the moduli spaces of stable bundles of rank two and determinant one, with low values of the second Chern class, on quintic surfaces. This study, which is joint work with Nicole Mestrano, will bring into play and illustrate many techniques: obstructions viewed as K_X-valued Higgs fields, the relationship with the geometry of linear systems on curves in $\mathbf{P}^3$, O'Grady's method of degeneration to the boundary via elementary transformations, and Hirschowitz's Methode d'Horace.