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Research Interest:

- Algebraic Geometry
- Arithmetic Geometry
- Diophantine Geometry

Research Summary:

The study of rational solutions to a system of polynomial equations with rational coefficients is called as *Diophantine geometry* whose origin dates back at least to the age of Greece. For example, Fermat's last theorem claims that a naive equation $x^n + y^n = z^n$ possesses only trivial solutions when $n \geq 3$, and this theorem has been proved by Wiles using highly advanced mathematics.

A modern approach in Diophantine geometry is to consider rational solutions as points on a geometric object called an *algebraic variety* which is defined by a system of polynomial equations, and this is a reason why rational solutions are called as *rational points*. My research mainly involves applying recent advances of higher dimensional algebraic geometry to problems in Diophantine geometry and applying the perspective of arithmetic geometry to problems in algebraic geometry.

My research has been centered around Manin's conjecture which is a conjectural asymptotic formula for the counting function of rational points on a Fano variety and the asymptotic formula is expressed in terms of birational invariants of the underlying variety. I have been studying birational geometric aspects of Manin's conjecture using higher dimensional algebraic geometry, and I also have been applying techniques from analytic number theory to prove Manin's conjecture for certain homogeneous spaces. So far my research can be summarized in three categories:

- We proposed a conjectural description of exceptional sets appearing in Manin's conjecture and proved that it is a thin set using the minimal model program. ([1], [7])
- We applied the above study of birational geometry of Manin's conjecture to problems on moduli spaces of rational curves on Fano varieties. ([2], [5], [8], [9])
- We proved Manin's conjecture and its variants for certain homogeneous spaces using the method of height zeta functions. ([4], [6])

Recently I am interested in homological stability of moduli spaces of rational curves and the motivic version of Manin's conjecture.

Major Publications:

[1] B. Lehmann and S. Tanimoto, On the geometry of thin exceptional sets in Manin's conjecture, Duke Math. J. **166** (2017), no. 15, 2815–2869,

[2] B. Lehmann and S. Tanimoto, Geometric Manin's conjecture and rational curves, Compos. Math. **155** (2019), no. 5, 833–862,

[3] S. Tanimoto, On upper bounds of Manin type, Algebra Number Theory **14** (2020), no. 3, 731–761,

[4] D. Loughran, R. Takloo-Bighash, and S. Tanimoto, Zero-loci of Brauer group elements on semi-simple algebraic groups, J. Inst. Math. Jussieu, **19** (2020), no. 5, 1467–1507,

[5] B. Lehmann and S. Tanimoto, Rational curves on prime Fano threefolds of index 1, J. Algebraic Geom., **30** (2021), no. 1, 151–188,

[6] M. Pieropan, A. Smeets, S. Tanimoto, and A. Várilly-Alavarado, Campana points of bounded height on vector group compacrifications, Proc. Lond. Math. Soc, **123** (2021), no. 1, 57–101,

[7] B. Lehmann, A. K. Sengupta, and S. Tanimoto, Geometric consistency of Manin's conjecture, Compos. Math. **158** (2022), no. 6, 1375–1427

[8] B. Lehmann and S. Tanimoto, Classifying sections of del Pezzo fibrations, I, to appear in J. Eur. Math. Soc. (JEMS),

[9] B. Lehmann, E. Riedl, and S. Tanimoto, Non-free sections of Fano fibrations, preprint,

Education and Appointments:

2012	Courant Institute of Mathematical Sciences, New York University, Ph.D.
2012	G.C. Evans Instructor, Department of Mathematics, Rice University
2015	PostDoc, Department of Mathematical Sciences, the University of Copenhagen
2018	Associate Professor, Priority Organization for Innovation and Excellence, Kumamoto University
2021	Associate Professor, Graduate School of Mathematics, Nagoya University

Message to Prospective Students:

Undergraduate students need to study the foundation of algebraic geometry, i.e., schemes and cohomology, and I have been using [2] for my seminar. To understand these, students need to be familiar with commutative algebra, and this can be acquired by reading [1] or [3].

I want my MS students to study higher dimensional algebraic geometry. Possible topics are: (1) the minimal model program (2) Positivity of divisors (3) Theory of rational curves. To understand (1), [4] is a standard textbook, but this book does not contain recent advances in the MMP such as BCHM. These advances can be studied by reading original papers after completing [4]. To learn (2), [5] are best textbooks. I have not read these recently and I am interested in reading them again. For (3), [6] is a good textbook. [7] is a more serious book, but it is a bit challenging to read.

MS students in their second year and Doctor students should conduct their own research. Topics of my recent students are: (1) Manin's conjecture for Campana points, (2) Moduli spaces of rational curves on Fano threefolds (3) Examples of exceptional sets in Manin's conjecture. Any topics in algebraic geometry and arithmetic geometry are welcome, so let me know if you have any problem you are interested in.

[1] M. F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra,

[2] R. Hartshorne, Algebraic Geometry, Springer

[3] S. Bosch, Algebraic Geometry and Commutative Algebra, Springer

[4] J. Kollár and Sh. Mori, Birational Geometry of Algebraic Varieties, Cambridge

[5] R. Lazarsfeld, Positivity in Algebraic Geometry I, II, Springer

[6] O. Debarre, Higher Dimensional Algebraic Geometry, Springer

[7] J. Kollár, Rational curves on Algebraic Varieties, Springer