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Membership of academic societies:
Mathematical Society of Japan

Research Interest:

- analytic methods in complex geometry
- geometric analysis

Research Summary:

1. Study of Hamiltonian volume minimizing property of Lagrangian submanifolds in Kähler-Einstein manifolds.

Recently I proved that any maximal torus orbit in the complex projective space is volume minimizing under Hamiltonian deformation. I proved this using the Borthwick-Paul-Uribe theory of semi-classical approximation of Lagrangian submanifolds satisfying the Bohr-Sommerfeld condition. However, the proof by geometric flow method would be more natural. The relevant geometric flow will be a higher dimensional generalization of the area preserving curve shortening flow on the sphere. This flow, which I like to call "Hamiltonian mean curvature flow", is an evolution equation which deforms a given Lagrangian submanifold to the derection corresponding to $\alpha - H(\alpha)$ where α is the mean curvature 1-form and $H(\alpha)$ is its harmonic part. I am now studying this geometric flow with my graduate students.

2. Study of Hamiltonian Kähler-Ricci flow. Its reduction to the complex Monge-Ampère flow. Toward a scalar curvature version of Calabi-Yau theorem and algebro-geometric stability of polarozed manifolds.

An analogous problem arises in Kähler geometry, i.e., the evolution equation deforming a given Kähler form ω in the direction of $-\mathrm{Ric}(\omega) + H(\mathrm{Ric}(\omega))$, where $\mathrm{Ric}(\omega)$ is the Ricci form and $H(\mathrm{Ric}(\omega))$ is its harmonic part w.r.to ω. This generalizes the Kähler-Ricci flow, which deforms a given Kähler form in $c_1(M) > 0$ to the direction $-\mathrm{Ric}(\omega) + \omega$. Recently, one of my graduate students and I was able to generalize the Futaki invariant as an obstruction to the existence of self-similar solutions to the Hamiltonian Kähler-Ricci flow. The proof implies that the Hamiltonian Kähler-Ricci flow equation (this includes constant scalar curvature Kähler metric as a special case) is reduced to a complex Monge-Ampère flow. Almost all fundamental properties of the Hamiltonian Kähler-Ricci flow (HKF) are yet to be studied. The idea of HKF will be applied to the so-called Donaldson-Tian-Yau conjecture on the equivalence of the existence of a cscK metric ω_{SF} and the K-stability of a polarized manifold. For instance, given a polarized manifold Y, let us consider a projective manifold X having Y as a divisor at infinity and try to find a scalar-flat complete Kähler metric on $X \setminus Y$. Then we can ask what is the relationship between the K-stability of Y and the behavior at infinity of the metric ω_{SF}. I an now studying this kind of problem concerning HKF with my graduate students.

3. Nevanlinna-Galois theory of algebraic minimal surfaces.

I am studying the value distribution theory of the Gauss map of algebraic minimal surfaces. The point of this theory is to establish relationship between Nevanlinna theory on the universal cover, i.e., the unit disk, and the action of the fundamental group. This is a joint work with Reiko Moyaoka (Tohoku University).

Major Publications:

[1] S. Bando and R. Kobayashi,"Ricci-flat Kähler metrics on affine algebraic manifolds, II", Math. Ann., **287** (1990), 175 – 180.

[2] R. Kobayashi, "Ricci-flat Kähler metrics on affine algebraic manifolds and degeneration of Kähler-Einstein K3 surfaces", Adv. Stud. Pure Math., **18-2** (1990), 137 – 228.

[3] Y. Kawakami, R. Kobayashi and R. Miyaoka, "The Gauss map of pseudo-algebraic minimal surfaces", Forum Mathematicum **20-6** (2008), 1055 – 1069.

[4] R. Kobayashi, "Toward Nevanlinna-Galois theory of algebraic minimal surfaces", in Riemann Surfaces, Harmonic Maps and Visualization, OCAMI STUDIES **3** (2010) 129 – 136.

[5] R. Kobayashi, "Ricci flow and geometrization conjecture (in Japanese)', Baifukan, (2011).

Awards and Prizes:

- Geometry award (1994)

Education and Appointments:

1988	PhD (University of Tokyo)
1983	Assistant Professor, Tohoku University
1988	Associate Professor, University of Tokyo
1992	Associate Professor, Nagoya University
1994	Professor, Nagoya University

Message to Prospective Students:

The field of my interest is geometric analysis. In particular, analytic methods in complex geometry. Geometric analysis may not be so familiar for undergraduate students or even for graduate students in early stage. Roughly speaking, analysis of various singular behaviors is most interesting and important in this field. Students who are enthusiastic in unifying various mathematics (including physical interpretations) are invited to this field. Here are examples of projects taken in my graduate seminar for PhD students: 1. Value distribution theory of the Gauss map of algebraic minimal surfaces (see research summary **3**), 2. Ricci flow and Harnack inequality, 3. "Hamiltonian mean curvature flow" (see research summary **1**), 4. "Hamiltonian Kähler-Ricci flow" (see research summary **2**).

The following books and papers are used in the above mentioned seminar :

[1] H. Fujimoto, Value distribution theory of the Gauss map of minimal surfaces in $\mathbb{R}^n$, Vieweg, 1993.

[2] B. Andrews and C. Baker, Mean curvature flow of pinched submanifolds to spheres, J. Diff. Geom. 85-3 (2010) 357-396.

[3] P. Topping, Lectures on the Ricci flow, http://www.maths.warwick.ac.uk/ topping/RFnotes.html.

[4] B: Kleiner and J. Lott, Notes on Perelman's papers, math.DG/0605667.

[5] G. Tian and X. Zhu, Convergence of Kähler-Ricci flow, J. Amer. Math. Soc. 20-3. (2007) 675-699.