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Membership of academic societies:
Mathematical Society of Japan, American Mathematical Society

Research Interest:

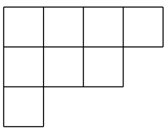
- Enumerative and Algebraic Combinatorics
- Combinatorial Representation Theory

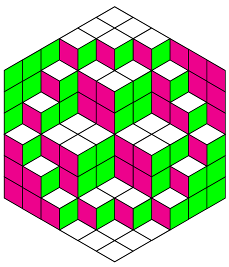
Research Summary:

My research interests are in combinatorics and its connection with algebra, representation theory, and integrable systems. I am working in the area where combinatorics meets with other fields of mathematics (and science). More specifically, I am interested in combinatorial problems arising from the representation theory of classical groups and related algebras, and also in enumeration problems of plane partitions, alternating sign matrices, and so on.

Combinatorial objects and techniques play an important role in representation theory. For example, Young diagrams (see the left figure below) are used to parameterize the irreducible representations of symmetric groups and general linear groups. Many concrete problems in representation theory (e.g., construction of irreducible representations, irreducible decomposition of a given representation) can be solved or attacked by using combinatorial methods. On the other hand, some combinatorial algorithms, e.g., the Robinson–Schensted correspondence, are now interpreted in terms of the crystal basis for quantum groups from the view point of representation theory. With these interactions between combinatorics and representation theory in mind, I study symmetric functions (characters and their generalizations) and determinant/Pfaffian identities ([2]).

Plane partitions are certain arrays of non-negative integers, which can be visualized as a stack of unit cubes (see the middle figure below). And alternating sign matrices are certain matrices with entries 1, 0 and −1 (see the right figure below), which are a generalization of permutation matrices. These objects were defined with purely combinatorial motivation, but they turned out to have relations with representation theory, statistical physics and so on. In my research, I enumerate certain classes of plane partitions, alternating sign matrices and related combinatorial objects by revealing hidden algebraic structures ([3]). In particular, I am interested in mysterious relationship between alternating sign matrices and totally symmetric self-complementary plane partitions.





$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Major Publications:

[1] S. Okada, Algebras associated to the Young–Fibonacci lattice, Trans. Amer. Math. Soc., **346** (1994), 549 – 568.

[2] S. Okada, Applications of minor summation formulas to rectangular-shaped representations of classical groups, J. Algebra **205** (1998), 337 – 367.

[3] S. Okada, Enumeration of symmetry classes of alternating sign matrices and characters of classical groups, J. Algebraic Combin. **23** (2006), 43 – 69.

[4] S. Okada, "Representation Theory of Classical Groups and Combinatorics", Baifukan, 2006 (in Japanese).

[5] M. Ishikawa and S. Okada, Identities for determinants and Pfaffians, and their applications, Sugaku **62** (2010), 85–114 (in Japanese).

Education and Appointments:

1990 Doctor of Science, University of Tokyo

1990 Research Associate, Nagoya University

1995 Associate Professor, Nagoya University

2006 Professor, Nagoya University

Message to Prospective Students:

I think that combinatorics is one of the most active and interesting areas in mathematics. Combinatorial problems or structures can be found in many branches of mathematics (and science), and the methods of algebraic combinatorics are applicable to these problems. So it is important to take interest not only in combinatorics but also in other related fields.

The interested students can start their study in combinatorics and its connection to algebra and representation theory with one of the following books.

1. R. P. Stanley, "Enumerative Combinatorics", Vol. 1 (2nd ed.), Vol. 2, Cambridge Univ. Press, 2012, 1999.
2. W. Fulton, "Young Tableaux : With Applications to Representation Theory and Geometry", Cambridge Univ. Press, 1996.
3. I. G. Macdonald, "Symmetric Functions and Hall Polynomials", Oxford Univ. Press, 1995.
4. J. Hong and S.-J. Kang, "Introduction to Quantum Groups and Crystal Bases", Amer. Math. Soc., 2002
5. A. Björner and F. Brenti, "Combinatorics of Coxeter groups", Springer, 2005.

Basic knowledge of linear algebra and abstract algebra is a necessary prerequisite for most of these books. Also the students can start their research activities in combinatorics at a relatively early stage.