OHTA, Hiroshi Professor



Office: Rm 325 in Sci. Bldg. A

Telephone: +81 (0) 52 - 789 - 2543 (ext. 2543)

E-mail: ohtaZmath.nagoya-u.ac.jp (Please replace Z by @)

Membership of academic societies:

Mathematical Society of Japan

Research Interest:

- symplectic geometry
- Floer theory
- gauge theory

Research Summary:

I am working on symplectic geometry, whose origin goes back to classical mechanics. Symplectic manifold is by definition a smooth manifold admitting a non-degenerate closed 2-form. Typical examples are cotangent bundle on which classical mechanics is described, and submanifolds in complex projective spaces.

My recent interest is Floer theory and relationship between singularities and symplectic/contact geometry. I am now working on Floer theory from the point of view of certain homotopical algebra, so called A_{∞} algebra. Such a homotopical algebra is a classical object originally arising from topology but it is now making new progress, partially motivated from physics. In particular, collaborating with K. Fukaya, Y-G. Oh and K. Ono, I constructed a filtered A_{∞} algebra associated to a Lagrangian submanifold of a symplectic manifold and developed Lagrangian intersection Floer theory based on the filtered A_{∞} algebra. (See Reference 1-[1] below.) This A_{∞} algebra plays an important and fundamental role in mirror symmetry, which claims correspondence between symplectic geometry on a symplectic manifold X and complex geometry on the mirror complex manifold X. As a consequence, for example, certain symplectic invariant of X defined by using solutions to some non linear partial differential equation will be surprisingly derived from certain complex geometric invariants of X defined by some linear differential equation. Our theory gives not only mathematical foundation in mirror symmetry but also provides some new applications to concrete problems in symplectic geometry. (See Reference 1-[2][3], for example.)

Major Publications:

- 1. Floer theory and mirror symmetry:
- [1] K. Fukaya, Y-G. Oh, H. Ohta and K. Ono, Lagrangian intersection Floer theory –Anomaly and Obstruction–. vol **46-1**, vol **46-2**. AMS/IP Studies in Advanced Mathematics. American Mathematical Society/International Press (2009).
- [2] K. Fukaya, Y-G. Oh, H. Ohta and K. Ono, Lagrangian Floer theory on compact toric manifolds
 I. Duke Math. J. 151, 23-175. (2010).
- [3] K. Fukaya, Y-G. Oh, H. Ohta and K. Ono, Lagrangian Floer theory on compact toric manifolds II: Bulk deformations. Selecta Math. New Series, 17, 609-711. (2011).
- [4] K. Fukaya, Y-G Oh, H. Ohta and K. Ono, Lagrangian Floer theory and mirror symmetry on compact toric manifolds. Astérisque, **376**, Société Mathématique de France (2016).

- 2. Singularity and symplectic/contact geometry:
- [1] H. Ohta and K. Ono, Simple singularities and topology of symplectically filling 4-manifold. Comment. Math. Helv. **74**. 575–590. (1999).
- [2] H. Ohta and K. Ono, Simple singularities and symplectic fillings. J. Differential Geom. **69**, 1–42. (2005).
- [3] H. Ohta and K. Ono, Examples of isolated surface singularities whose links have infinitely many symplectic fillings. J. Fixed Point Theory and Applications. 3, (V.I. Arnold Festschrift Volume) 51–56. (2008).
- 3. Gauge theory:
- [1] M. Furuta and H. Ohta, Differentiable structures on punctured 4-manifolds. Topology and its Appl. **51**. 291–301 (1993).
- [2] H. Ohta and K. Ono, Notes on symplectic 4-manifolds with $b_2^+ = 1$, II. Internat. J. of Math. 7. 755–770. (1996).
- [3] H. Ohta, Brieskorn manifolds and metrics of positive scalar curvature. Advance Studies Pure Math. **34**. 231–236. (2002).

Message to Prospective Students:

Here are some examples of texts which I used in my seminar (for the first year of master course).

- 1. M. Audin, Torus actions on symplectic manifolds, 2nd revised edition, Birkhäuser (2004).
- 2. M. Audin and M. Damian, Morse theory and Floer homology, Springer. (2014).
- 3. D. McDuff and D. Salamon, Introduction to symplectic topology, Oxford Univ. Press (1995).
- 4. N. Hitchin, The self-dual equations on a Riemann surface, Proc. London Math. Soc **55** (1987) 59-126.
- 5. H. Hofer and E. Zehnder, Symplectic invariants and Hamiltonian dynamics, Birkhäuser. (1994).

It is expected to already master manifold theory, (co)homology theory, elementary differential geometry, topology but the most important is to study by yourself what you don't know. Of course, I will give some advice and suggestion, if necessary. To get an impression on the basic literature, please look at the following books:

- 1. K. Fukaya, Symplectic geometry, Iwanami, (1999) (in Japanese).
- 2. D. McDuff and D. Salamon, J-holomorphic curves and symplectic topology, American Math. Soc. (2004).
- 3. P. Seidel, Fukaya categoryies and Picard-Lefscetz theory, Zurich Lectures in Advanced Math., Eurp. Math. Soc. (2008).