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### Research Interest:

- analytic methods in complex geometry
- geometric analysis

### Research Summary:

**1.** Study of the value distribution of the Gauss map of algebraic minimal surfaces :

This is a classical problem going back to Osserman's pioneering work in 1960's. The present status of this problem is that one cannot go further without some break through. After several year's intensive study of this problem, I reached a collective Cohn-Vossen inequality which is based on the localization principle arising from the iteration of a parabolic translation. I am now constructing the value distribution theory on the footing of the collective Cohn-Vossen inequality.

**2.** Study of the volume of Lagrangian submanifolds under Hamiltonian deformation in Kähler-Einstein manifolds :

The problem I address here is that of constructing a framework for understanding how the volume changes under Hamiltonian deformation of a maximal torus orbit in the complex projective space. So far, I observed that the micro-local approach based on Borthwick-Paul-Urbe theory is effective. As this problem goes back to a naive question "does any simple closed curve in  $S^2$  converge to a circle by the area preserving curve shortening flow?", this is to be studied by the method of geometric flow (Hamiltonian mean curvature flow).

**3.** Scalar-flat complete Kähler metric and Monge-Ampère equation. Scalar curvature version of the Calabi-Yau theorem and  $K$ -stability of polarized manifolds :

An analogous problem arises in Kähler geometry, i.e., the evolution equation deforming a given Kähler form  $\omega$  in the direction of  $-\text{Ric}(\omega) + H(\text{Ric}(\omega))$ , where  $\text{Ric}(\omega)$  is the Ricci form and  $H(\text{Ric}(\omega))$  is its harmonic part w.r.to  $\omega$ . This geometric flow will be applied to the so-called Yau-Tian-Donaldson conjecture on the equivalence of the existence of a cscKähler metric  $\omega_{\text{csc}}$  and the  $K$ -stability of a polarized manifold. For instance, given a polarized manifold  $Y$ , let us consider a projective manifold  $X$  having  $Y$  as a divisor at infinity and try to find a scalar-flat complete Kähler metric on  $X \setminus Y$ . Then we can ask what is the relationship between the  $K$ -stability of  $Y$  and the behavior at infinity of the metric  $\omega_{\text{SF}}$ . For a preparation of this problem, I am studying a singular perturbation of the Calabi-Yau theorem on Fano manifolds and its scalar curvature version, as well as an extension of Ding's functional to general polarization via variational methods introduced by Berman et al.

### Major Publications:

- [1] S. Bando and R. Kobayashi, "Ricci-flat Kähler metrics on affine algebraic manifolds, II", Math. Ann., **287** (1990), 175 – 180.

- [2] R. Kobayashi, “Ricci-flat Kähler metrics on affine algebraic manifolds and degeneration of Kähler-Einstein K3 surfaces”, *Adv. Stud. Pure Math.*, **18-2** (1990), 137 – 228.
- [3] Y. Kawakami, R. Kobayashi and R. Miyaoka, “The Gauss map of pseudo-algebraic minimal surfaces”, *Forum Mathematicum* **20-6** (2008), 1055 – 1069.
- [4] R. Kobayashi, “Toward Nevanlinna-Galois theory of algebraic minimal surfaces”, in *Riemann Surfaces, Harmonic Maps and Visualization*, *OCAMI STUDIES* **3** (2010) 129 – 136.
- [5] R. Kobayashi, “Ricci flow and geometrization conjecture (in Japanese)”, *Baifukan*, (2011).
- [6] R. Kobayashi and R. Miyaoka, “Nevanlinna-Galois theory for algebraic and pseudo-algebraic minimal surfaces – value distribution of the Gauss map –”, preprint (2015).

### Awards and Prizes:

- Geometry award (1994)

### Education and Appointments:

- 1988 PhD (University of Tokyo)
- 1983 Assistant Professor, Tohoku University
- 1988 Associate Professor, University of Tokyo
- 1992 Associate Professor, Nagoya University
- 1994 Professor, Nagoya University

### Message to Prospective Students:

The field of my interest is geometric analysis. In particular, analytic methods in complex geometry. Geometric analysis may not be so familiar for undergraduate students or even for graduate students in early stage. Roughly speaking, analysis of various singular behaviors is most interesting and important in this field. Students who are enthusiastic in unifying various mathematics (including physical interpretations) are invited to this field. Here are examples of projects taken in my graduate seminar for PhD thesis :

1. Projective embedding of Abelian varieties and Lagrangian fibration.
2. Geometric analysis of higher dimensional Gauge theory.
3. Value distribution theory of the Gauss map of algebraic minimal surfaces.
4. Ricci flow and Lie groups.
5. Ricci flow and Harnack inequality.
6. Existence problem for Kähler-Ricci solitons under certain special situation. Convergence problem of modified Kähler-Ricci flow under general polarization under certain special situation.

The following books and papers are used in the above mentioned seminar :

- [1] H. Fujimoto, Value distribution theory of the Gauss map of minimal surfaces in  $\mathbb{R}^n$ , Vieweg, 1993.
- [2] B. Andrews and C. Baker, Mean curvature flow of pinched submanifolds to spheres, *J. Diff. Geom.* 85-3 (2010) 357-396.
- [3] P. Topping, Lectures on the Ricci flow, <http://www.maths.warwick.ac.uk/topping/RFnotes.html>.
- [4] B. Kleiner and J. Lott, Notes on Perelman’s papers, *math.DG/0605667*.
- [5] G. Tian and X. Zhu, Convergence of Kähler-Ricci flow, *J. Amer. Math. Soc.* 20-3. (2007) 675-699.