HISHIDA, Toshiaki Professor

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Membership of academic societies:
Mathematical Society of Japan

Research Interest:

- Nonlinear partial differential equations
- Navier-Stokes equation

Research Summary:

My research field is nonlinear partial differential equations (PDEs) arising from fluid mechanics. The study is mostly based on a framework of functional analysis with the aid of harmonic analysis, however, detailed hard analysis of each equation by use of own mathematical structure leads us to deeper results on qualitative properties of solutions such as regularity, stability and asymptotic behavior. Such efforts for various equations could provide insight about new development of the theory of PDEs.

The motion of the fluid is essentially nonlinear because of its deformation. On the other hand, even at the level of linear analysis such as linearized equation around a certain steady state, we often find a difficult structure. Furthermore, several properties of solutions depend on the fluid region, such as exterior domain, aperture domain and so on. On account of the deep aspect they say nowadays mathematical fluid mechanics is a fascinating branch of PDEs.

Among several PDEs arising from fluid mechanics, the Navier-Stokes system

$$\partial_t u + u \cdot \nabla u = \Delta u - \nabla p, \qquad \text{div } u = 0$$

which describes the motion of a viscous incompressible fluid is well known. It was derived in the 19th century from conservations of momentum and mass together with constitutive equation. The unknown physical quantities are the velocity $u(x,t)$ and pressure $p(x,t)$ of the fluid at position x and time t, and the problem of finding them under initial and boundary conditions is called initial-boundary value problem. On the boundary of the obstacle, we usually impose the no-slip condition ($u = 0$ when the obstacle is at rest) due to the viscosity. As the boundary condition at space infinity for, say, the exterior problem, we often assume that the flow is at rest or tends to a prescribed constant velocity. Mathematical analysis of the Navier-Stokes system is traced back to a series of celebrated papers by Leray in 1930s. Later on, remarkable progress has been made by a lot mathematicians and it has always had much influence on analysis of some other PDEs. In spite of efforts for 80 years since the landmark by Leray, however, the following problem still remains open: The unique existence of regular solution globally in time without any smallness assumption on initial data no matter how smooth they are. Besides this, many other challenging problems also exist; in particular, it is of utmost importance to find mathematical features of flows in various regions arising in physically relevant situation, such as the flow in the exterior of a rotating obstacle ([2], [3], [4], [6]) and the flow through an aperture ([5]).

Major Publications:

[1] T. Hishida, On the relation between the large time behavior of the Stokes semigroup and the decay of steady Stokes flow at infinity, Progr. Nonlinear Diff. Equat. Appl. **60** (2011), 343–355.

[2] R. Farwig and T. Hishida, Asymptotic profile of steady Stokes flow around a rotating obstacle, Manuscripta Math. **136** (2011), 315–338.

[3] T. Hishida and Y. Shibata, L_p-L_q estimate of the Stokes operator and Navier-Stokes flows in the exterior of a rotating obstacle, Arch. Rational Mech. Anal. **193** (2009), 339–421.

[4] R. Farwig and T. Hishida, Stationary Navier-Stokes flow around a rotating obstacle, Funkcial. Ekvac. **50** (2007), 371–403.

[5] T. Hishida, The nonstationary Stokes and Navier-Stokes flows through an aperture, *Contributions to Current Challenges in Mathematical Fluid Mechanics*, 79–123, Adv. Math. Fluid Mech., Birkhäuser, Basel, 2004.

[6] T. Hishida, An existence theorem for the Navier-Stokes flow in the exterior of a rotating obstacle, Arch. Rational Mech. Anal. **150** (1999), 307–348.

Awards and Prizes:

- Analysis Prize (2007)

Education and Appointments:

1993 Dr. Sci., Waseda University
1993 Research Associate, Waseda University
1994 Research Associate, Kumamoto University
1997 Assistant Professor, Niigata University
2000 Associate Professor, Niigata University
2008 Professor, Nagoya University

Message to Prospective Students:

As the subject in the seminar of master course, I can propose, for instance, (1) elliptic PDEs of second order; (2) the method of functional analysis such as semigroup theory; (3) mathematical analysis of the Navier-Stokes system, which are related each other. If he/she studies continuously with me for two years, he/she can proceed from (1) and (2) to (3). As the textbook, I can recommend

1. L. C. Evans, Partial Differential Equations, Amer. Math. Soc., 1998.
2. D. Gilbarg and N. S. Trudinger, Elliptic Partial Differential Equations of Second Order, Springer, 1977.
3. H. Sohr, The Navier-Stokes Equations, An Elementary Functional Analytic Approach, Birkhäuser, 2001.
4. G. P. Galdi, An Introduction to the Mathematical Theory of the Navier-Stokes Equations, Second Edition, Springer, 2011.

For those who wish to proceed to the doctor course, they are asked to read some related papers. In the doctor course, what is important is to find a nice problem and to develop analysis by himself/herself. It might be better to work on a bit different subject from mine.