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Research Interest:

- Complex Geometry
- Constant scalar curvature Kähler metric

Research Summary:

We are studying geometry of algebraic varieties in view of complex-analysis, using for example $\bar{\partial}$ -equation or Monge–Ampère equation. In recent the problem of constant scalar curvature Kähler metric much attracts me. You know, any smooth plane cubic is homeomorphic to $\mathbb{S}^1 \times \mathbb{S}^1$ and its complex structure is uniformized by \mathbb{C} . It implies that any smooth cubic admits a flat curvature Kähler metric. What we are involved in is the higher dimensional case of such a phenomenon. That is, when a given variety embedded in a projective space admits a constant scalar curvature Kähler metric? In higher dimension, smoothness is not sufficient to assure the existence of the special metric. On the contrary, we have to degenerate the manifold to singular varieties! Such degeneration is intensively studied in geometric invariant theory or say, moduli theory of algebraic varieties. People expect that an appropriate stability notion coming from this area gives a necessary and sufficient condition for the existence of constant scalar curvature Kähler metric. For Fano manifolds the conjecture is in fact proved by Tian and Chen–Donaldson–Sun in 2012. General case is however still very open and I am interested in the problem in view of complex analysis. The spirit of the conjecture originates from the equivariant version of Atiyah–Singer index theorem so that the asymptotic analysis of Bergman kernels gives us a way to understand this problem. When I was a Ph.D student, I refined such Bergman kernel asymptotics to apply it to the problem of constant scalar curvature Kähler metrics. One result gives an analytic description of a norm-like invariant which measures a size of a degeneration. Now we are showing that such a norm plays an essential role in measuring how stable a given manifold is.

Major Publications:

- [1] T. Hisamoto: On the volume of graded linear series and Monge–Ampère mass. *Math. Z.* **275** (2013), no. 1-2, 233–243.
- [2] T. Hisamoto: On the limit of spectral measures associated to a test configuration of a polarized Kähler manifold, *J. Reine Angew. Math.* DOI: 10.1515/crelle-2014-0021, April 2014.

Education and Appointments:

- 2013 JSPS Post-doc at Nagoya University
- 2015 Assistant Professor, Nagoya University

Message to Prospective Students:

One of attractions of complex geometry is that it's on a crossroad of differential geometry, complex analysis, and algebraic geometry. Students who like an analytic proof of an algebraic statement (as Dirichlet's theorem on arithmetic progressions?) would take a fancy to it. To get an impression on the basic literature, please look at the following books. If you got interested in the area, please don't hesitate to ask.

- Complex geometry

First of all, you should experience Riemann surfaces (or algebraic curves in view of algebraic geometry). There are a lot of textbooks which include

O. Forster: Lectures on Riemann Surfaces.

If you got interested in the higher dimension, I recommend

S. Kobayashi: Complex geometry (written in Japanese).

In higher dimension terms of differential geometry, *e.g* connection and curvature, will appear.

- Several complex variables:

You need several complex variables in studying algebraic varieties from analytic point of view. There are many topics in several complex variables but I here introduce a textbook of L^2 -theory which is much compatible with Bergman kernel.

R. Hörmander: An Introduction to Complex Analysis in Several Variables.

A rapid course to learn applications of those techniques to algebraic varieties is

J. P. Demailly: Analytic Methods in Algebraic Geometry.

The last part of the book is devoted to an introduction to Demailly's recent research.

- Algebraic geometry:

R. Hartshorne: Algebraic Geometry

is a standard textbook of scheme. As I said in the above, one needs to consider singular schemes even if he started from smooth one. If you like to go further and study geometric invariant theory,

D. Mumford: Geometric Invariant Theory

is very detailed especially in the relation with symplectic geometry and equivariant index theorem.