Office: Rm 304 in Math. Bldg.
Telephone: +81 (0)52–789–4877 (ext. 4877)
E-mail: hideyuki@math.nagoya-u.ac.jp
Website: http://www.math.nagoya-u.ac.jp/~hideyuki/
Membership of academic societies:
Mathematical Society of Japan

Research Interest:

- Geometry and analysis on a homogeneous space which is not necessarily symmetric
- Representation theory of a Lie group which is not necessarily reductive

Research Summary:

A positive definite symmetric matrix is often regarded as a natural multi-dimensional analogue of a positive number. For instance, recall that the Hessian matrix of a function of several variables plays the same roles as the second derivative of a function of one variable does. Let us consider the set $\mathrm{Sym}^+(n,\mathbf{R})$ of positive definite symmetric matrices of size n, which is a multi-dimensional analogue of the half line $\mathbb{R}_+$. We can obtain various integral formulas about $\mathrm{Sym}^+(n,\mathbf{R})$, and the integrals play important roles in statistics (Wishart), analytic number theory (Siegel), and theory of partial differential equation (Gårding) eventually. Here we note that the shape of $\mathrm{Sym}^+(2,\mathbf{R})$ is exactly the circular cone. Indeed, the matrix $\begin{pmatrix} z+x & y \\ y & z-x \end{pmatrix}$ is positive definite if and only if $z > \sqrt{x^2+y^2}$. For a general n, the set $\mathrm{Sym}^+(n,\mathbf{R})$ forms an open convex cone in $\mathbb{R}^{n(n+1)/2}$, on which the group $GL(n,\mathbb{R})$ acts linearly and transitively in the way $x \mapsto gx\,{}^tg$ $(x \in \mathrm{Sym}^+(n,\mathbf{R}), g \in GL(n,\mathbb{R}))$. It is due to this action why the integral formulas over $\mathrm{Sym}^+(n,\mathbf{R})$ holds. In general, an open convex cone is called a homogeneous cone if a Lie group acts on the cone linearly and transitively. Since I was a graduate student, the fascinating homogeneous cones have been in the center of my research interest. What luck I met them!

Symmetric cones are special instances of homogeneous cones. They have particularly rich structure due to an action of a reductive Lie group, and many mathematicians study the cones thoroughly. Like the Platonic solid, they are quite beautiful and precious. On the contrary, a general theory of homogeneous cones has been developed not so much since Vinberg and Gindikin established fundamental works about them. Most mathematicians including Gindikin himself have thought that we may not expect so much result about general homogeneous cones because of the lack of their symmetry. For the same reason, a little number of researchers are working on the theory of homogeneous (not necessarily symmetric) bounded domain nowadays, though they were studied by many authors before.

However, even if the spaces are not so tidy, the calculus over them can be carried out tidily. If it is the case, the situation should be very interesting, shouldn't it? And my research experience convinces me that it is truly the case. The crucial fact is that every homogeneous cone is obtained as an intersection of $\mathrm{Sym}^+(n,\mathbf{R})$ and an appropriate vector subspace, where the group action is also described in terms of matrices. Based on my new theory, I dream of unifying the existing works on homogeneous cones by many authors and practice further calculations over the cones, which will supply interesting examples to complex geometry, differential geometry, representation theory of Lie group, theory of prehomogeneous vector space and automorphic form, theory of partial differential equation, and statistics.

Major Publications:

[1] H. Ishi and C. Kai, The representative domains of a homogeneous bounded domain, Kyushu J. Math. **64** (2010), 35–47.

[2] H. Ishi, On symplectic representations of normal j-algebras and their application to Xu's realizations of Siegel domains, Differential Geom. Appl. **24** (2006), 588–612.

[3] H. Ishi, Wavelet transforms for semidirect product groups with not necessarily commutative normal subgroups, J. Fourier Anal. Appl. 12 (2006), 37–52.

[4] H. Ishi, Positive Riesz distributions on homogeneous cones, J. Math. Soc. Japan **52** (2000), 161–186.

Awards and Prizes:

- 2000, Takebe Prize (Mathematical Society of Japan), 'Analysis on homogeneous cones and homogeneous Siegel domains'

Education and Appointments:

2000 Research assistant, Yokohama City University

2007 Associate Professor, Nagoya University

Message to Prospective Students:

The subject of my seminar is representation theory of Lie group. Our aim is to appreciate the relations between geometry and analysis on homogeneous spaces. Since we get the information about the geometry of a space from functions over the space, and vice versa, in modern mathematics, it is quite natural to grasp the symmetry of a space by observing the representation of the transformation group realized on function spaces over the original space. In particular, many formulas about special functions such as recurrence formulas, differential equations and definite integrals are understood clearly from the viewpoint of representation theory, which reflects the symmetry of the space deeply. Moreover, the interplay of the symmetry of a space and group representation plays fundamental roles in quantum mechanics. Keeping such perspective in mind, we shall read one of the following references in the seminar.

[1] T. Kobayashi and T. Oshima, Lie Groups and Representations, Iwanami, 2005 (in Japanese).

[2] D. P. Želobenko, Compact Lie groups and their representations, Translations of Mathematical Monographs **40**, American Mathematical Society, 1973.

[3] N. Ja. Vilenkin, Special functions and the theory of group representations, Translations of Mathematical Monographs **22**, American Mathematical Society, 1968.

[4] G. B. Folland, Harmonic analysis in phase space, Annals of Mathematics Studies **122**, Princeton University Press, 1989.

[5] S. T. Ali, J.-P. Antoine, J.-P. Gazeau, Coherent states, wavelets and their generalizations, Graduate Texts in Contemporary Physics, Springer, 2000.

The students of my seminar are assumed to have knowledge about elementary group theory as well as calculus and linear algebra, and they should learn more advanced knowledge such as theory of manifolds and functional analysis as the need arises.