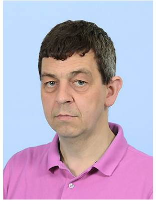
HESSELHOLT, Lars Professor



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Membership of academic societies:

American Mathematical Society
European Mathematical Society
Danish Mathematical Society

Research Interest:

- Algebraic K-Theory
- Equivariant Homotopy Theory
- p-Adic Arithmetic Geometry

Research Summary:

My area of specialization is algebraic K-theory and algebraic topology, in general, and topological cyclic homology and equivariant stable homology theory, in particular. As homological algebra came about because of the fact that not every module is projective, algebraic K-theory came about because of the fact that not every projective module is free. That is, homological algebra and algebraic K-theory are necessary in order to do linear algebra over a ring that is not a field. Such rings appear in all brances of mathematics, e.g. the coordinate ring of scheme in algebraic geometry and number theory and the integral and spherical group rings of a discrete group in geometric topology.

Despite the name, algebraic K-theory does not admit a purely algebraic definition. Instead, following Quillen, the algebraic K-groups of a ring R are defined to be the homotopy groups

$$K_n(R) = \pi_n(K(R))$$

of a (pointed) topological space $K(R)$ built by gluing together simplices in a way that reflects the structure of the category of finitely generated projective right R-modules. To understand the structure of these groups is a very deep problem indeed. For instance, the statement that $K_{4m}(\mathbb{Z}) = 0$ for all positive integers m is equivalent to the Kummer-Vandiver conjecture in number theory which states that no prime number p divides the class number of the field $\mathbb{Q}(\zeta_p + \zeta_p^{-1})$. (The conjecture is known to hold for $p < 163,000,000$.) The cyclotomic trace map, which is a generalization of the classical Chern character, is a natural map from algebraic K-theory to a topological refinement of Connes' cyclic homology defined by Bökstedt-Hsiang-Madsen. It is an important tool for understanding algebraic K-theory. Indeed, for non-regular rings, it is currently the only tool available. For instance, it is proved in the paper [1] that if $\mathcal{O}_K$ a complete discrete valuation ring with quotient field $K = \mathcal{O}_K[1/p]$ of characteristic 0 and algebraically closed residue field $k = \mathcal{O}_K/\mathfrak{m}_K$ of odd characteristic p, then there is the following canonical isomorphism.

$$K_*(K, \mathbb{Z}/p\mathbb{Z}) \to (W\Omega^*_{\mathcal{O}_K}(\log \mathfrak{m}_K) \otimes S_{\mathbb{Z}/p\mathbb{Z}}(\mu_p))^{F=1}$$

To formulate this result, it was necessarly to define the de Rham-Witt complex with log poles that appear on the right-hand side. In this way, understanding the structure of algebraic K-theory often necessitates the creation of new mathematics. Motivic cohomology is another example of new mathematics that was created in this way.

Major Publications:

[1] L. Hesselholt and I. Madsen, On the K-theory of local fields, Ann. of Math. **158** (2003), 1–113.

[2] T. Geisser and L. Hesselholt, The de Rham-Witt complex and p-adic vanishing cycles, J. Amer. Math. Soc. **19** (2006), 1–36.

[3] T. Geisser and L. Hesselholt, Bi-relative algebraic K-theory and topological cyclic homology, Invent. Math. **166** (2006), 359–395.

[4] L. Hesselholt, On the p-typical curves in Quillen's K-theory, Acta Math. **177** (1996), 1–53.

[5] L. Hesselholt and I. Madsen, Cyclic polytopes and the algebraic K-theory of truncated polynomial algebras, Invent. Math. **130** (1997), 73–97.

Awards and Prizes:

- Alfred P. Sloan Fellowship (1998)
- Foreign Member of the Royal Danish Academy of Sciences and Letters (2012)
- Niels Bohr Professor (2013–2018)
- Clay Senior Scholar (2014)

Education and Appointments:

1994 Institut Mittag-Leffler Postdoctoral Fellow
1994 C.L.E. Moore Instructor, M.I.T.
1997 Assistant Professor, M.I.T.
2001 Associate Professor, M.I.T.
2008 Professor, Nagoya University

Message to Prospective Students:

The research area of algebraic K-theory is suitable for students with strong interest and background in algebra, number theory, and homotopy theory. It is an area that intersects numerous mathematical fields and for that reason it is necessary to learn a lot of background material before starting cutting-edge research. For that reason, it is important that students are capable of reading books and research papers independently. Here is a representative list of books and articles:

[1] D. Quillen, Homotopical algebra, Lecture Notes in Math. 43, Springer, Berlin 1967.

[2] D. Quillen, Higher algebraic K-theory. I. *Algebraic K-theory, I: Higher K-theories (Proc. Conf., Battelle Memorial Inst., Seattle, WA, 1972)*, pp. 85–147. Lecture Notes in Math., 341, Springer, Berlin 1973.

[3] F. Waldhausen, Algebraic K-theory of spaces. *Algebraic and geometric topology (New Brunswick, N.J., 1983)*, 318?419, Lecture Notes in Math., 1126, Springer, Berlin, 1985.

[4] J. Milnor, Introduction to algebraic K-theory. Annals of Mathematics Studies, 72. Princeton University Press, Princeton, N.J., 1971.