The cyclotomic Temperley-Lieb algebras

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Cyclotomic Temperley-Lieb algebras $TL_{n,m}$ are a class of subalgebras of cyclotomic Brauer algebras. They are cellular in the sense of Graham and Lehrer. The cell modules are indexed by the set

$$\{(k, \mathbf{i}) \mid \mathbf{i} = (i_1, i_2, \cdots, i_{n-2k}), 1 \le i_j \le m, 0 \le k \le [n/2]\}.$$

Using standard results on cellular algebras, we can classify its irreducible modules over a splitting field of $x^m - 1$. We also give a necessary and sufficient condition for $TL_{n,m}$ to be semi-simple. Explicitly, $TL_{n,m}$ is semi-simple if and only if all cell modules with respect to $(1, (i_1, i_2, \cdots, i_{n-2}))$ are non-isomorphic irreducible (not all k). This implies that the semi-simplicity of $TL_{n,m}$ is determined by a class of polynomials, called generalized Tchebychev polynomials.

This is based on two papers by Xi-R and Xi-Yu-R.