WORKSHOP

GEOMETRIC ANALYSIS IN GEOMETRY AND TOPOLOGY 2014

Date : October 28th – 31th, 2014
Place : Morito Memorial Hall
Tokyo University of Science
6-3-1 Niijuku, Katsushika-ku, Tokyo 125-8585, JAPAN

Invited speakers (Survey lecturers)
- **Pierre Albin** (University of Illinois at Urbana-Champaign, USA)
- **Gilles Carron** (Université de Nantes, France)
- **Justin Corvino** (Lafayette College, USA)
- **Tom Mrowka** (MIT, USA)
- **Daniel Ruberman** (Brandeis Univ., USA)

Program

October 28th (Tue.)

10:00–11:00
Daniel Ruberman (Brandeis Univ.)
" Index theory on end-periodic manifolds—End-periodic differential operators "

11:30–12:30
Tom Mrowka (MIT)
" Invariants for Knots and Webs from Singular Instantons I "

12:30–14:00 **Lunchtime**

14:00–15:00
Justin Corvino (Lafayette College)
" Deformation and gluing constructions for scalar curvature, with applications I "

15:30–16:30
Pierre Albin (University of Illinois at Urbana-Champaign)
" Hodge theory on singular spaces I "

October 29th (Wed.)

10:00–11:00
Daniel Ruberman (Brandeis Univ.)
“ Index theory on end-periodic manifolds—Applications of the end-periodic index theorem ”

11:30–12:30
Tom Mrowka (MIT)
“ Invariants for Knots and Webs from Singular Instantons II ”

12:30–14:00 **Lunchtime**

14:00–15:00
Gilles Carron (Université de Nantes)
“ Survey on some geometrical estimates of the Heat and Green kernel on non compact complete manifolds ”

15:30–16:30
Justin Corvino (Lafayette College)
“ Deformation and gluing constructions for scalar curvature, with applications II ”

18:00 ∼ **Dinner (convivial party)**

October 30th (Thu.)

10:00–11:00
Daniel Ruberman (Brandeis Univ.)
“ Index theory on end-periodic manifolds—Examples and extensions ”

11:30–12:30
Tom Mrowka (MIT)
“ Invariants for Knots and Webs from Singular Instantons III ”

12:30–14:00 **Lunchtime**

14:00–15:00
Gilles Carron (Université de Nantes)
“ Harmonic analysis and the Riesz transform on non compact complete Riemannian manifolds with quadratic (Ricci) curvature decay ”

15:30–16:30
Pierre Albin (University of Illinois at Urbana-Champaign)
“ Hodge theory on singular spaces II ”

October 31th (Fri.)

10:00–11:00
Justin Corvino (Lafayette College)
" Deformation and gluing constructions for scalar curvature, with applications III"

11:30–12:30
Pierre Albin (University of Illinois at Urbana-Champaign)
" Hodge theory on singular spaces III "

12:30–14:00 **Lunchtime**

14:00–15:00
Gilles Carron (Université de Nantes)
" Harmonic functions and the topology of non compact complete Riemannian manifolds "

Abstract

- **Pierre Albin** : **Hodge theory on singular spaces.**

Abstract The cohomology of any smooth closed manifold can be represented analytically as the de Rham group of closed forms modulo exact forms. If the manifold has a Riemannian metric, then in each cohomology class we can find a unique harmonic representative. On singular spaces the situation is more complicated. If the singularities are geometrically controlled, in that the space is 'stratified,' then there is an analogous story as long as the cohomology and the metric are adapted to the singularities. These spaces arise naturally when studying smooth spaces or maps, for instance, as algebraic varieties, orbit spaces or moduli spaces. The seminal work on these cohomologies is due to Goresky-MacPherson and Cheeger. I will report on joint work with Eric Leichtnam, Rafe Mazzeo, and Paolo Piazza extending and refining these theories to general stratified spaces.

- **Gilles Carron** :

アブストラクト Lecture I : **Survey on some geometrical estimates of the Heat and Green kernel on non compact complete manifolds.**

I will introduce the basics tools (Sobolev Inequality, Faber-Krahn inequality, Poincaré inequalities) that are useful in order to insure estimate of the heat kernel or the Green Kernel of a complete Riemannian manifold, following old idea of Nash and recent result inspired by Grigor'yan and Saloff-Coste.

Lecture II : **Harmonic analysis and the Riesz transform on non compact complete Riemannian manifolds with quadratic (Ricci) curvature decay.**

If (M^n, g) is a complete Riemannian manifold, the Riesz transform is the operator $d\Delta_g^{-\frac{1}{2}}$. The spectral theorem implied that the Riesz transform is bounded on L^2. In general, it is of interest to ?gure out the range of p for which the Riesz transform extends to a bounded operator on L^p. I will describe some results in this

direction on certain manifold with quadratic (Ricci) curvature decay.

Lecture III : **Harmonic functions and the topology of non compact complete Riemannian manifolds.**

When (M^n, g) is a complete Riemannian manifold with (at least) 2 ends, it is useful to know wether it is possible to find a harmonic function, with L^2 gradient that tends to 1 when one tends at infinity in one end and that tends to -1 when one tends at infinity in any other ends. I will describe when we can find such a harmonic function and give several applications of this idea.

- **Justin Corvino** : **Deformation and gluing constructions for scalar curvature, with applications.**

Abstract The scalar curvature function on a Riemannian manifold is the total trace of the Riemann curvature tensor, and manifests itself geometrically through the Taylor expansion of the volume contained inside geodesic spheres. The scalar curvature also arises in the context of the initial value problem for the Einstein equation, as a measure of the local energy density of the physical fields in certain initial data. This scalar metric invariant is generally very flexible, as indicated by the Kazdan-Warner classification of the space of scalar curvatures of metrics on closed manifolds, as illustrated by the Fischer-Marsden deformation theory, and as seen via conformal methods such as those used to settle the famous Yamabe problem. On the other hand, there are well known topological obstructions to positive scalar curvature, dating from the seminal works by Lichnerowicz, Gromov and Lawson, and Schoen and Yau, and tying in to the Positive Energy Theorem in general relativity.

In these lectures, we explore aspects of the geometry and analysis of the scalar curvature. In particular we discuss a construction of, and obstructions to, small prescribed compactly supported deformations of the scalar curvature via compactly supported metric deformations. Various applications and extensions of this localized scalar curvature deformation will be presented, as time permits. Such applications include constructions of initial data for the Einstein constraint equations with prescribed asymptotic structure; a recent gluing result (joint with M. Eichmair and P, Miao) for constant scalar curvature metrics with control on the total volume; and as an interesting application of this latter result, a recent resolution (by S. Matsuo) of the remaining cases of the modified Kazdan-Warner problem of prescribed scalar curvature with a volume constraint on closed Riemannian manifolds.

- **Tom Mrowka** : **Invariants for Knots and Webs from Singular Instantons.**

Abstract TBA

- **Daniel Ruberman** :

Abstract **Three Lectures on Index theory on end-periodic manifolds.** These lectures will survey ongoing work with Tom Mrowka and Nikolai Saveliev on the index theory of elliptic operators on non-compact manifolds with periodic ends.

Lecture I : **End-periodic differential operators.**
I will start with the basic topological setup, and discuss end-periodic differential operators. Examples come from diverse sources, including gauge theory, knot theory, and differential geometry. I will describe Taubes' criterion for an end-periodic differential operator to be Fredholm, and discuss some circumstances under which this holds. Then I will state our index theorem, which is analogous to the famous Atiyah-Patodi-Singer index theorem; the statement includes a new 'periodic' eta-invariant.

Lecture II : **Applications of the end-periodic index theorem.**
The periodic eta-invariant extends to a twisted version, analogous to the Atiyah-Patodi-Singer rho-invariant. Applying this to the Dirac operator, we give new classes of even-dimensional manifolds (including dimension 4) with disconnected moduli spaces of positive scalar curvature metrics. Time permitting, I will briefly describe how the end-periodic index theorem fits in with older work on Seiberg-Witten invariants for non-simply connected 4-manifolds.

Lecture III : **Examples and extensions.**
I will start with the extension of the index theorem to the setting where Taubes' Fredholm criterion fails to hold. An interesting example of this occurs in considering the de Rham complex (where the index is replaced by a suitable Euler characteristic), and I will describe how we calculate the Euler characteristic. The result, a version of the Gauss-Bonnet theorem, includes additional topological contributions coming from the Alexander polynomial of the periodic end.

Organizers
- Naoyuki Koike (Tokyo University of Science)
- Shu Nakamura (University of Tokyo)
- Mikio Furuta (University of Tokyo)
- Osamu Kobayashi (Osaka University)
- Shinichiroh Matsuo (Osaka University)
- Rafe Mazzeo (Stanford University, Foreign adviser)
- Kazuo Akutagawa (Tokyo Institute of Technology)