Experimental implementation of near-optimal quantum measurements of optical coherent states

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DEX-SMI Workshop on Quantum Statistical Inference, NII, 3 March 2009

Quantum optics: experimentally feasible approach to demonstrate quantum state discriminations

polarization (& location) encoding in single-photon states

Minimum error discrimination

Huttner *et al*., Phys. Rev. A 54, 3783 (1996)

Unambiguous state discrimination

Clarke *et al*., Phys. Rev. A 63, 040305(R) (2001)

Collective measurements

Fujiwara *et al*., Phys. Rev. Lett. 90, 167906 (2003)
Pryde et al., Phys. Rev. Lett. 94, 220406 (2005)

etc.....

encoding in coherent states

Programmable unambiguous state discriminator

Bartuskova *et al*., Phys. Rev. A 77, 034406 (2008)

⇨ **For applications?**

Original motivation for the state discrimination

Quantum Detection and Estimation Theory

Carl W. Helstrom

Department of Applied Physics
and Information Science
University of California, San Diego
La Jolla, California

ACADEMIC PRESS New York San Francisco London 1976

A Subsidiary of Harcourt Brace Jovanovich, Publishers

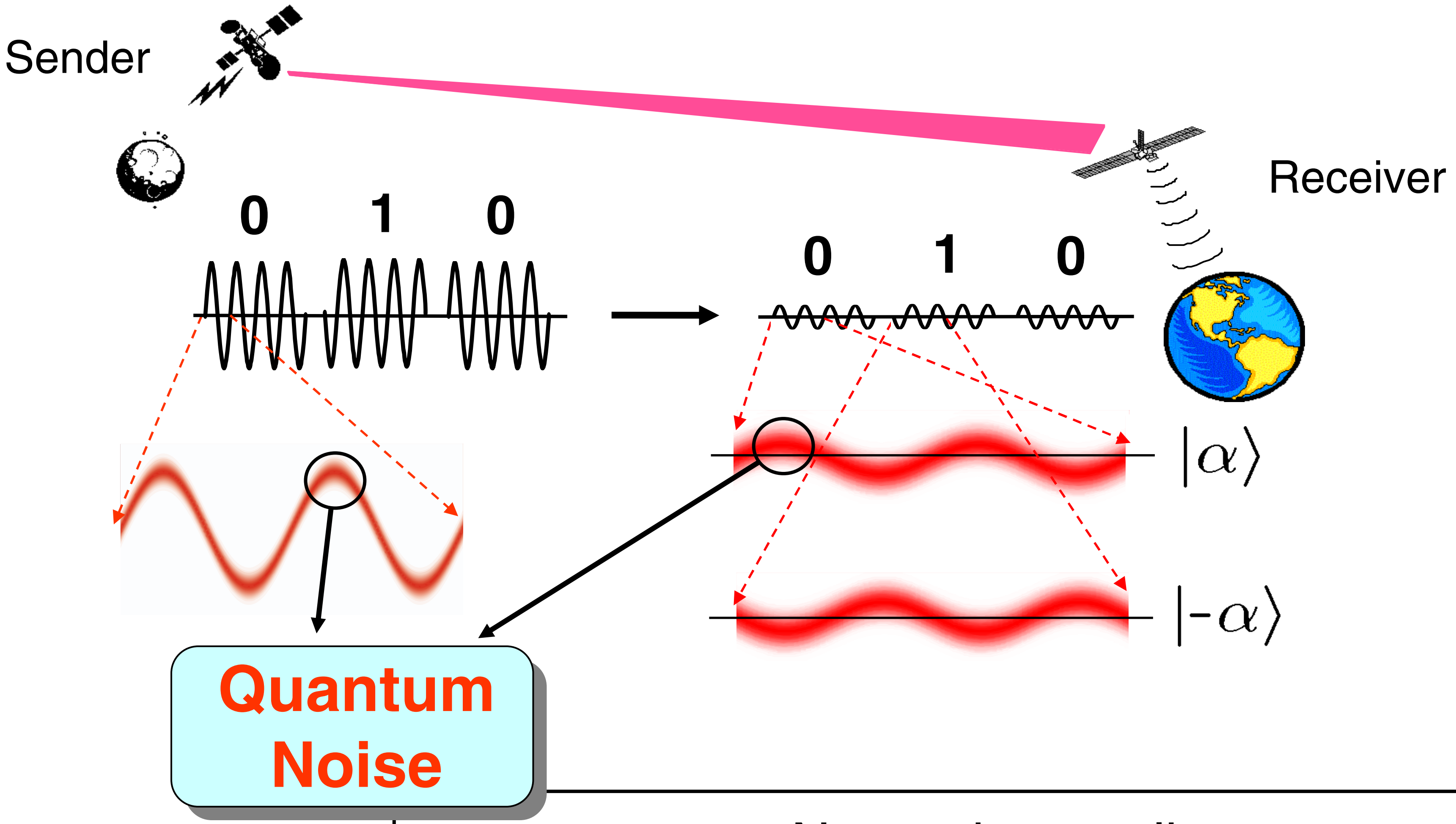
Preface

This book addresses two groups of readers. The first includes communications engineers and scientists and students of communication theory who need to cope with basic problems arising in communication with optical signals. The ultimate detectability of optical signals and the accuracy with which their parameters can be estimated cannot be ascertained by the methods of detection theory that apply at radio frequencies; the fundamental concepts of the theory must be revised, and this book shows how. The second group of readers comprises physicists interested in the foundations and applications of quantum mechanics, for whom it may be fruitful to consider quantum measurement as a process of decision among alternative density operators, or as estimation of certain parameters of the density operator of a quantum system. May they find the problems analyzed here a challenge to their conceptions of the quantum theory.

Those whose principal interest lies in optical communications may, at least on first reading, omit §3 of Chapter III, §1(d) of Chapter IV, §6 of Chapter V, and §2 of Chapter VIII. Quantum physicists may skim lightly over the details in Chapters VI and VII and in §§5 and 6 of Chapter VIII. References to the bibliography at the end of the book are coded with the authors' initials and the year of publication; thus [AlE 74] refers to a paper by Ali and Emch that appeared in 1974. For those who may wish to make a broader study of this subject, the bibliography contains a few papers not specifically cited in the text.

C. W. Helstrom 1976

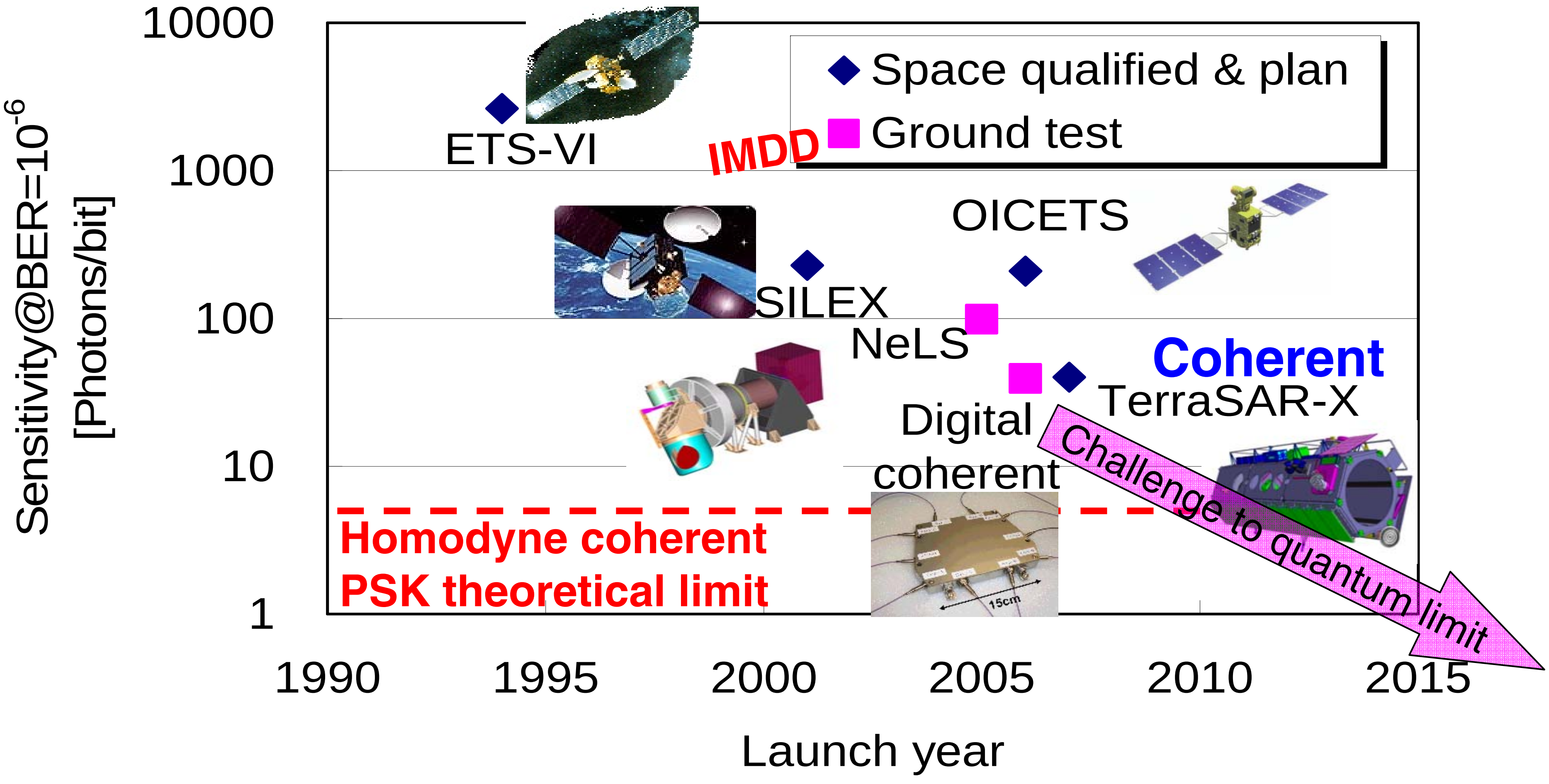
Quantum noise in optical coherent states



Non-orthogonality

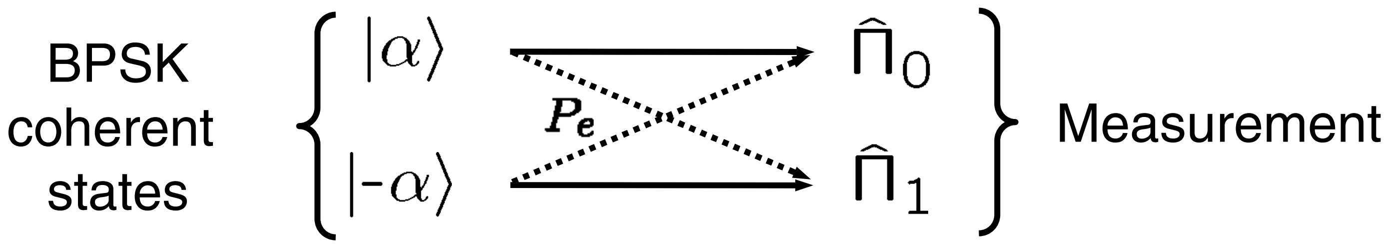
$$\langle\alpha|-\alpha\rangle = e^{-2|\alpha|^2} > 10^{-6} \text{ for } |\alpha|^2 < 3$$

Trends of optical receiver sensitivity



Discrimination of binary coherent states

Binary Coherent States: $\{|\alpha\rangle, |-\alpha\rangle\}$ $\langle\alpha|-\alpha\rangle = e^{-2|\alpha|^2}$



POVM

$$\sum \hat{\Pi}_i = \hat{I}$$

$$\hat{\Pi}_i \geq 0$$

Min. error discrimination

→ Projection onto the superpositions of coherent states

$$\hat{\Pi}_i = |\pi_i\rangle\langle\pi_i| \quad \begin{cases} |\pi_0\rangle = a|\alpha\rangle - b|-\alpha\rangle \\ |\pi_1\rangle = b|\alpha\rangle - a|-\alpha\rangle \end{cases}$$

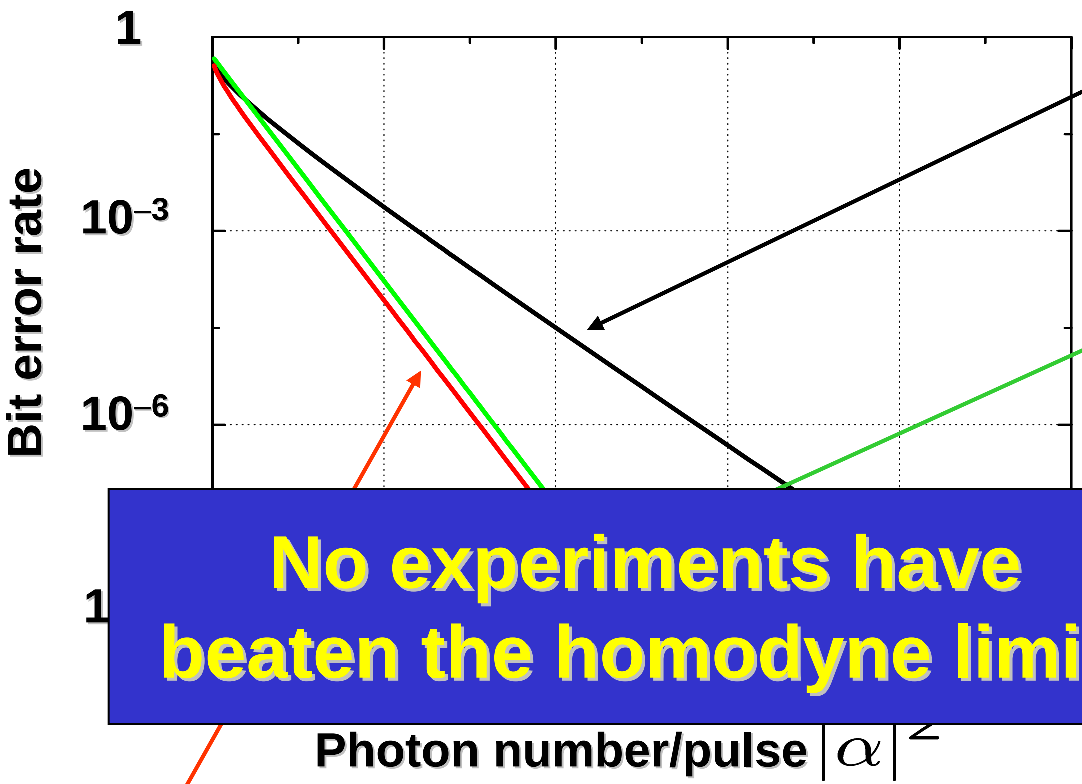
$$a = \sqrt{\frac{1+\sqrt{1-\kappa^2}}{2(1-\kappa^2)}}$$

$$b = \sqrt{\frac{1-\sqrt{1-\kappa^2}}{2(1-\kappa^2)}}$$

$$\kappa = |\langle\alpha| - \alpha\rangle|$$

Minimum Error Probability: $P_e = \frac{1}{2}\left(1 - \sqrt{1-\kappa^2}\right)$

Quantum receivers



No experiments have beaten the homodyne limit!

Homodyne limit

(Coherent optical communication)

Near optimal receiver (Kennedy receiver)

R. S. Kennedy, RLE, MIT, QPR, 108, 219 (1973)

- Coherent local oscillator
- Photon counter

Minimum error (Helstrom bound)

Optimal receiver (Dolinar receiver)

S. J. Dolinar, RLE, MIT, QPR, 111, 115, (1973)

- Coherent local oscillator
- Photon counter
- Classical feedback (infinitely fast!)

Contents

1. Homodyne measurement

⇨ The optimal strategy within Gaussian operations and classical communication

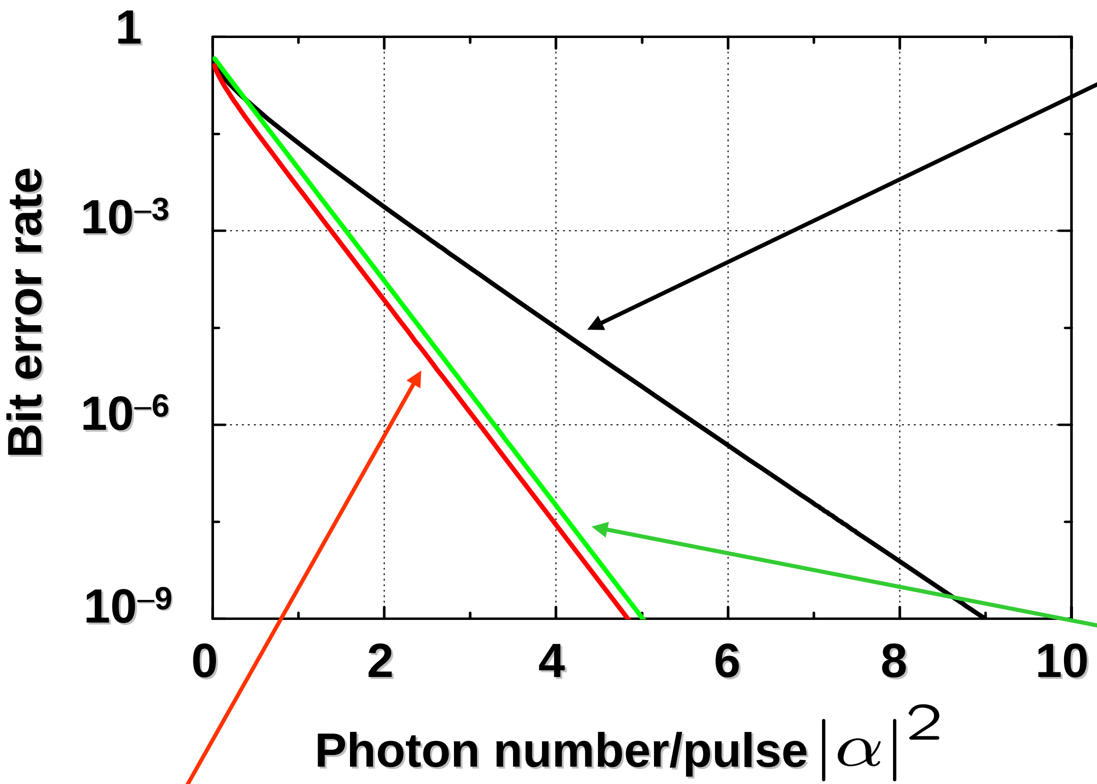
2. Practical near-optimal quantum receiver (Improvement of the Kennedy receiver)

2-1 Proposal and proof-of-principle experiment

Toward beating the homodyne limit:

2-2 Device: superconducting photon detector (TES)
2-3 Theory: performance evaluation via the cut-off rate

Quantum receivers



Homodyne limit (SNL)

(Coherent optical communication)

Best strategy within Gaussian operations and classical communication (feedback)

Kennedy receiver

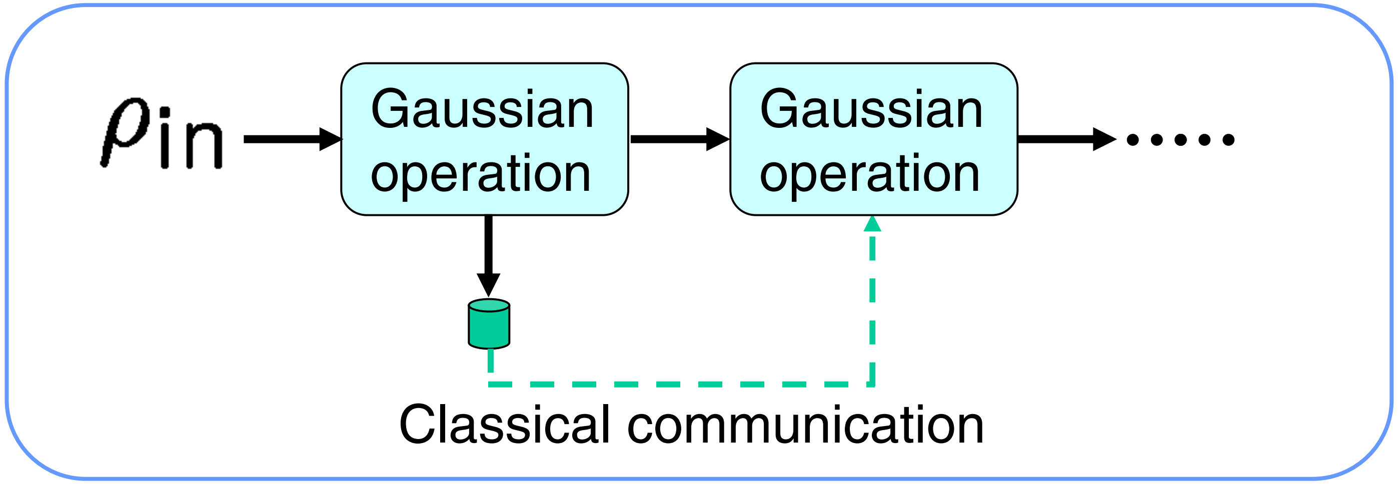
R. S. Kennedy, RLE, MIT, QPR, 108, 219 (1973)

Minimum error (Helstrom bound)

Dolinar receiver

S. J. Dolinar, RLE, MIT, QPR, 111, 115, (1973)

Gaussian operations and classical communication (GOCC)



If ρ_{in} is a Gaussian state,

⇨ any classical communication does not help the protocol!

(for any trace decreasing Gaussian CP map, one can construct a corresponding trace preserving GCP map)

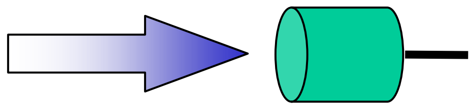
Eisert, et al, PRL 89, 137903 (2002)
Fiurasek, PRL 89, 137904 (2002)
Giedke and Cirac, PRA 66, 032316 (2002)

Gaussian operations and classical communication (GOCC)

In our problem, $|\alpha\rangle$ and $|-\alpha\rangle$ are Gaussian.

However, the receiver does not know which signal is coming..

$$\rho_{\text{in}} = p_+|\alpha\rangle\langle\alpha| + p_-|-\alpha\rangle\langle-\alpha|$$



Measurement via GOCC

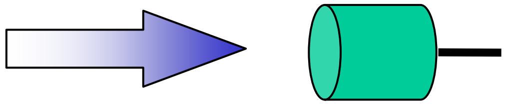
non-Gaussian state!

Does classical communication increase the distinguishability?

without CC

Discrimination via Gaussian measurement without CC.

$$p_+|\alpha\rangle\langle\alpha| + p_-|-\alpha\rangle\langle-\alpha|$$



Optimal measurement under Bayesian strategy…

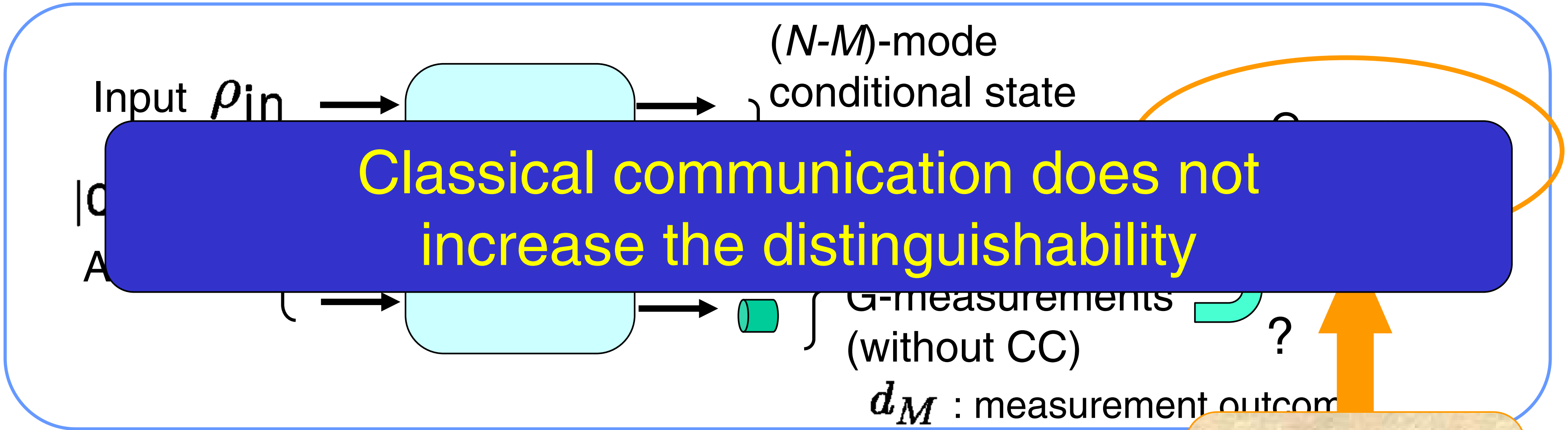
→ Homodyne measurement with $\varphi = 0$

(independent on $p_\pm$)

Average error probability

$$P_e^{(G)} = \frac{p_+}{2}\mathrm{erfc}\left[\sqrt{2}\alpha + \frac{\ln(p_+/p_-)}{4\sqrt{2}\alpha}\right] + \frac{p_-}{2}\mathrm{erfc}\left[\sqrt{2}\alpha - \frac{\ln(p_+/p_-)}{4\sqrt{2}\alpha}\right]$$

Classical communication (conditional dynamics)



$$\tilde{\rho}_{out} = p_+ \underline{\tilde{P}_+(d_M)} |\Psi_+^G\rangle\langle\Psi_+^G| + p_- \underline{\tilde{P}_-(d_M)} |\Psi_-^G\rangle$$

measurement-dependent

Homodyne measurement
$\varphi = 0$

$|\Psi_\pm^G\rangle$: pure Gauss…n states

$$\tilde{\rho}_{out} \xrightarrow{\hat{U}_G} \rho_{out} = \left(p_+ P_+(d_M)|\alpha'\rangle\langle\alpha'| + p_- P_-(d_M)|-\alpha'\rangle\langle-\alpha'|\right)$$

$$\otimes|0\rangle\langle 0|^{\otimes N-M-1}$$

Minimum error discrimination of binary coherent states under Gaussian operation and classical communication is achieved by a simple homodyne detection

Homodyne limit ⟶ Limit of Gaussian operations

Takeoka and Sasaki, Phys. Rev. A 78, 022320 (2008)

- For multiple coherent states? multi-partite signals?
- Classical-quantum capacity with restricted (GOCC) measurement?

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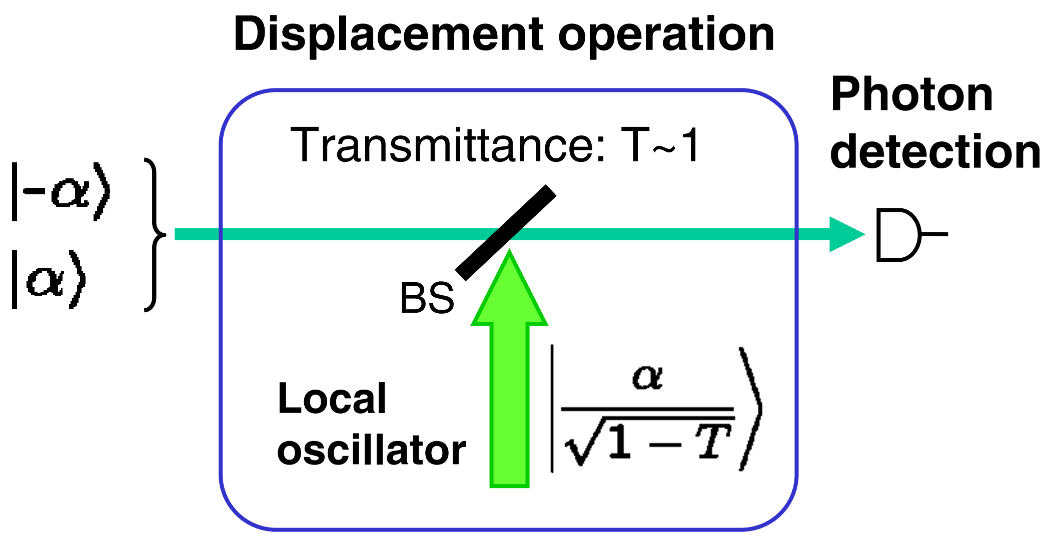
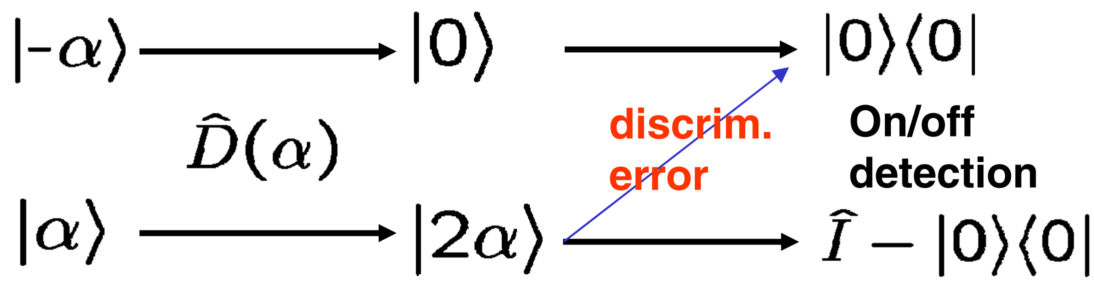
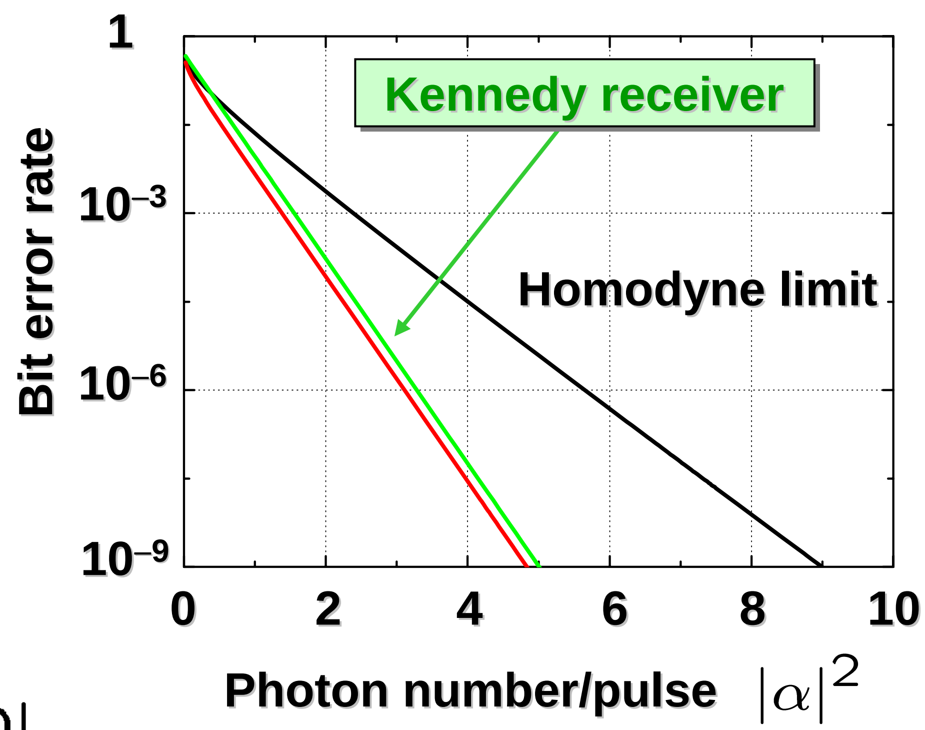
Kennedy receiver

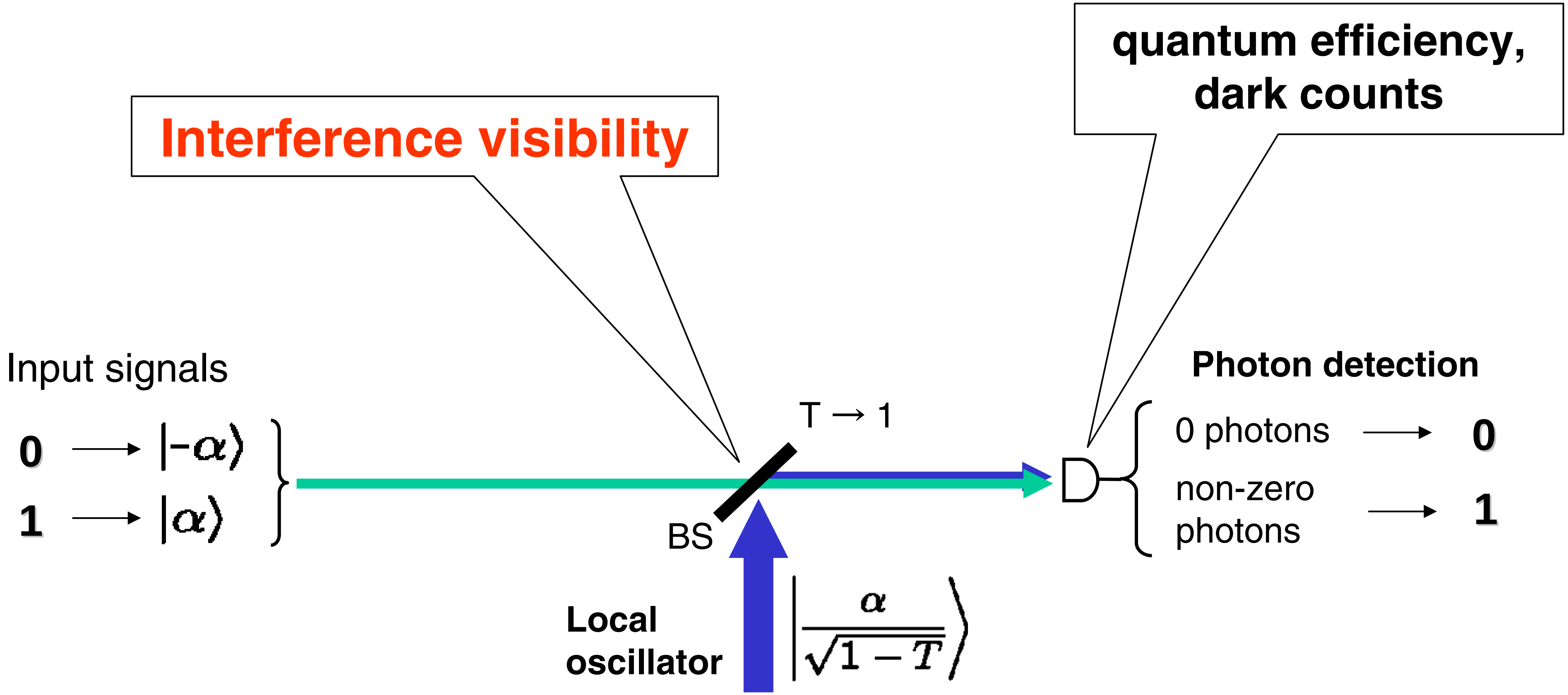
Kennedy, RLE, MIT, QPR 108, 219 (1973)



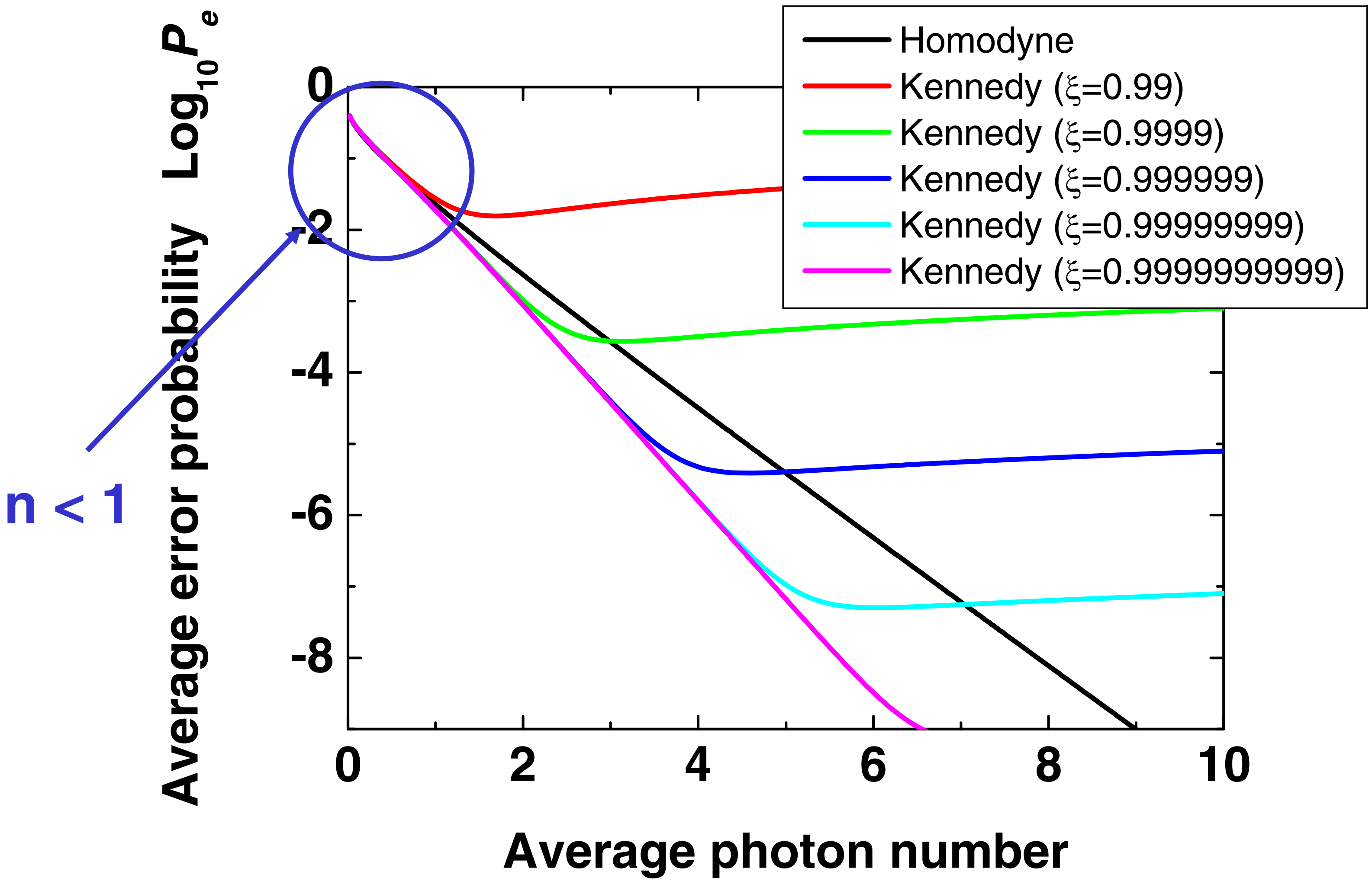




Practical imperfections

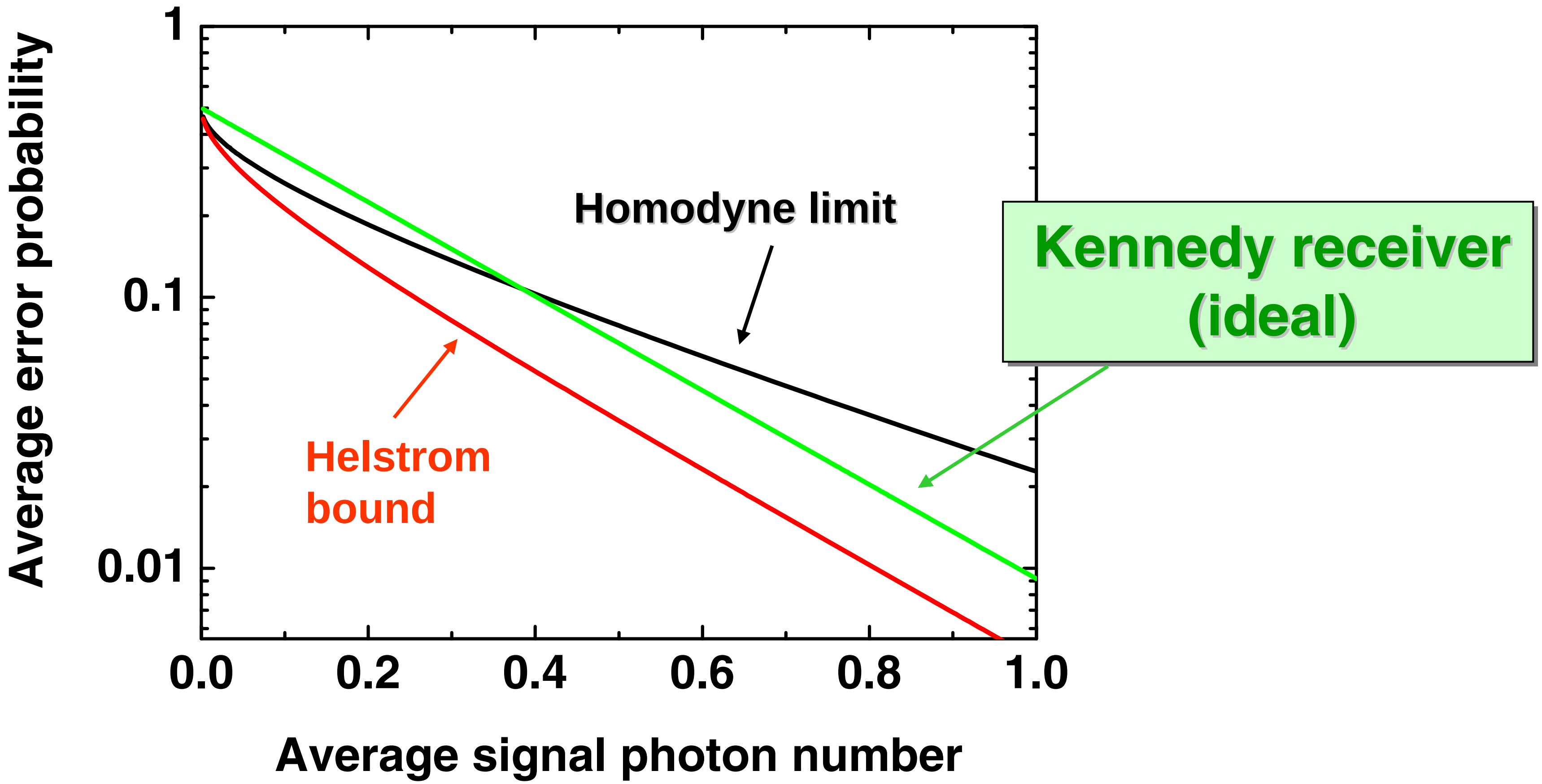


Visibility



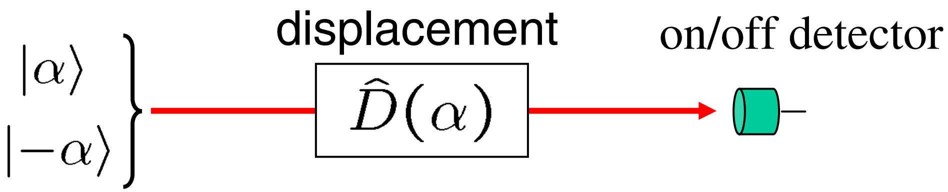
Homodyne
Kennedy (ξ=0.99)
Kennedy (ξ=0.9999)
Kennedy (ξ=0.999999)
Kennedy (ξ=0.99999999)
Kennedy (ξ=0.9999999999)
Average error probability Log10Pe
Average photon number
0
-2
-4
-6
-8
0
2
4
6
8
10
n < 1

Kennedy receiver at extremely weak signals

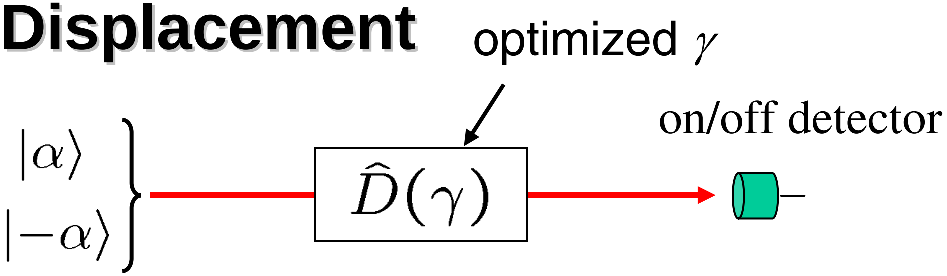


Generalizing of the Kennedy receiver

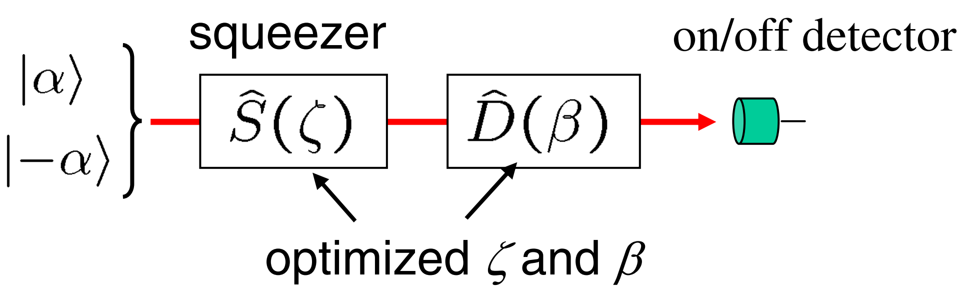
Kennedy receiver



Optimal Displacement

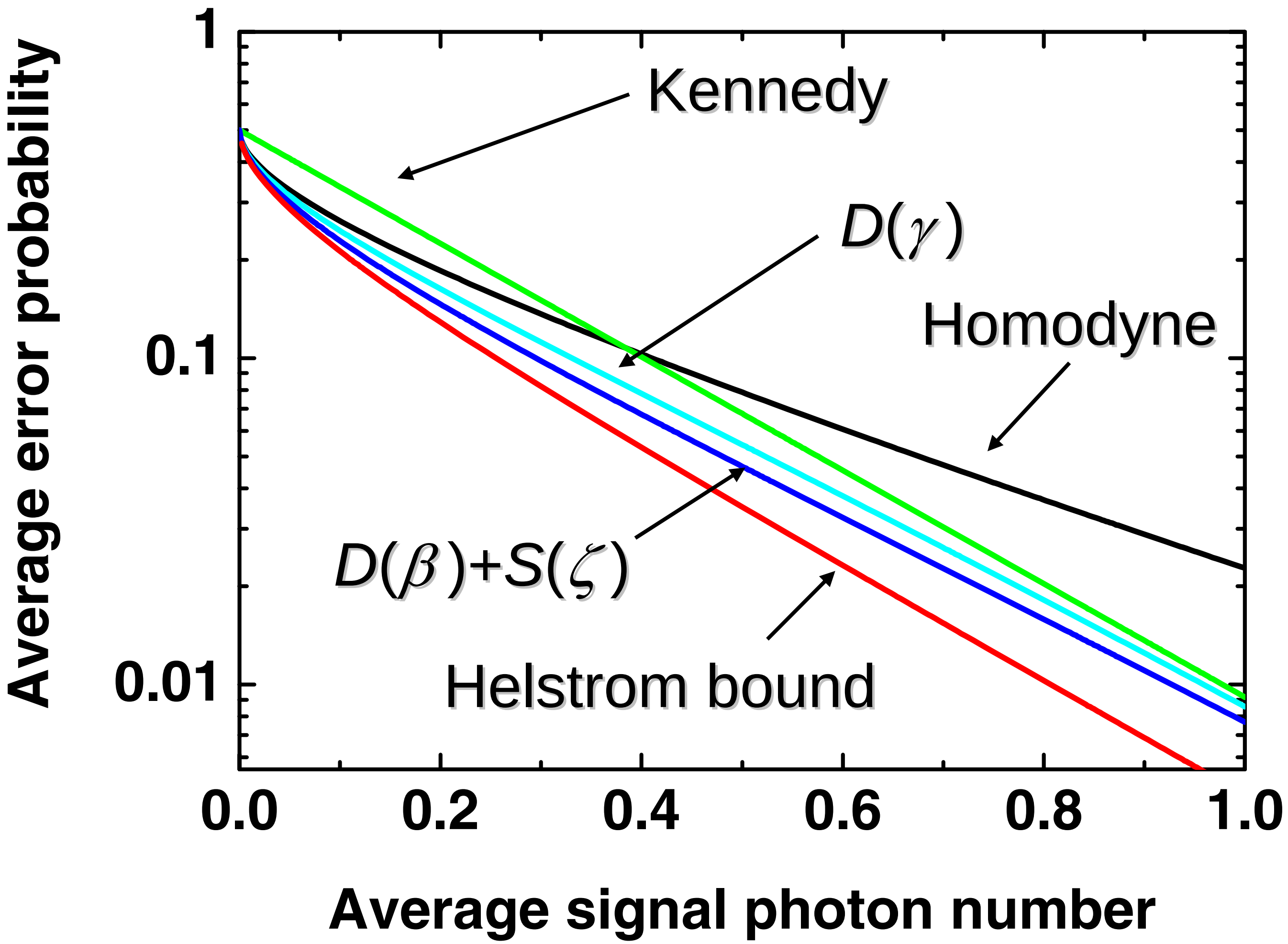


Squeezing + Displacement (Gaussian unitary operation)



Takeoka and Sasaki,
Phys. Rev. A 78, 022320 (2008)

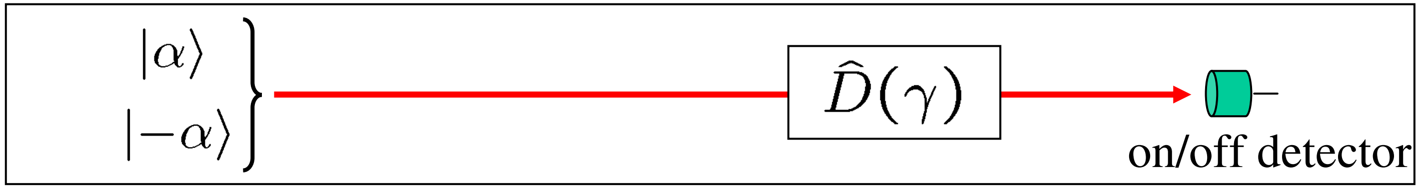
Average error probabilities

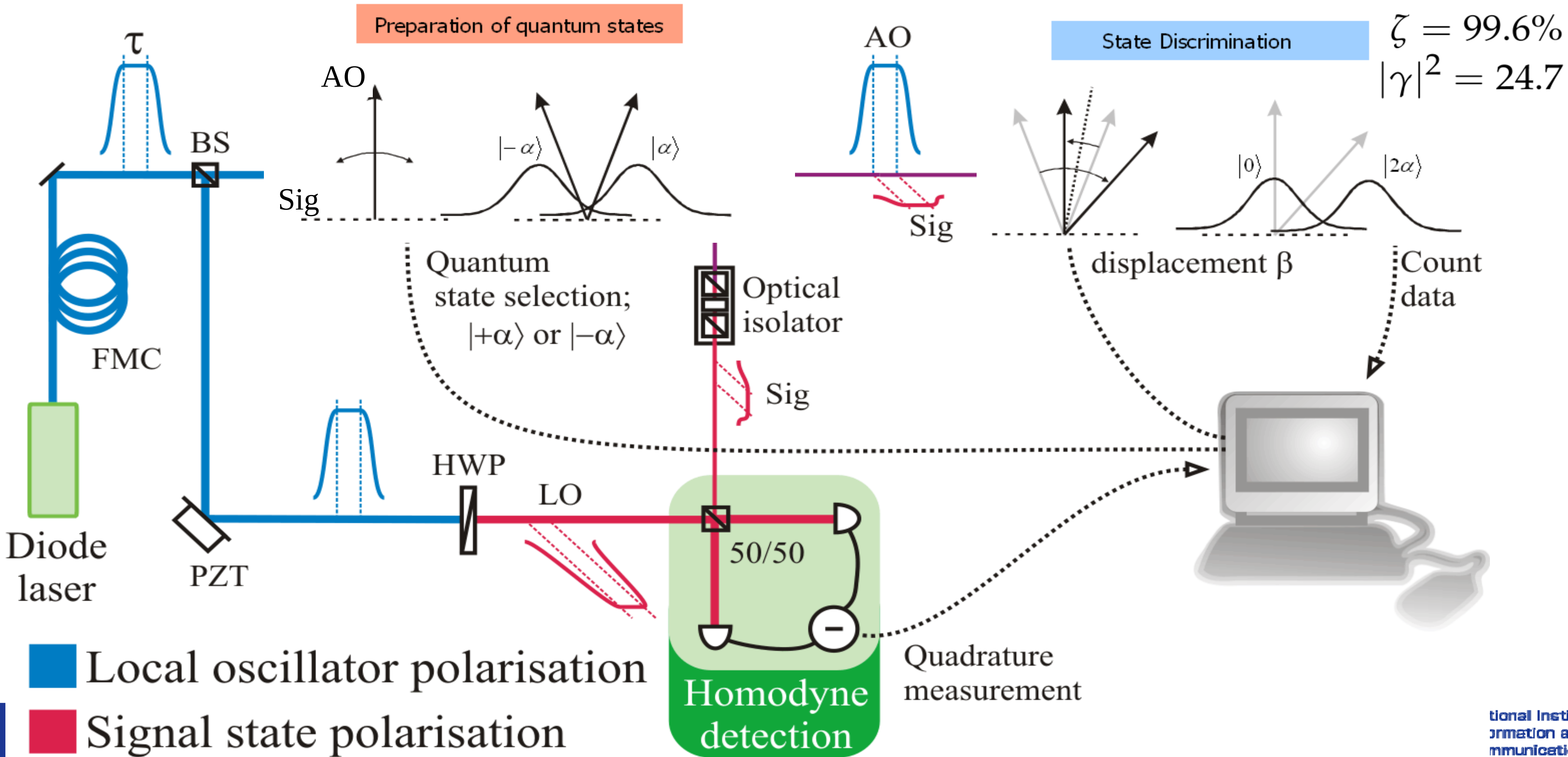


Proof-of-principle experiment

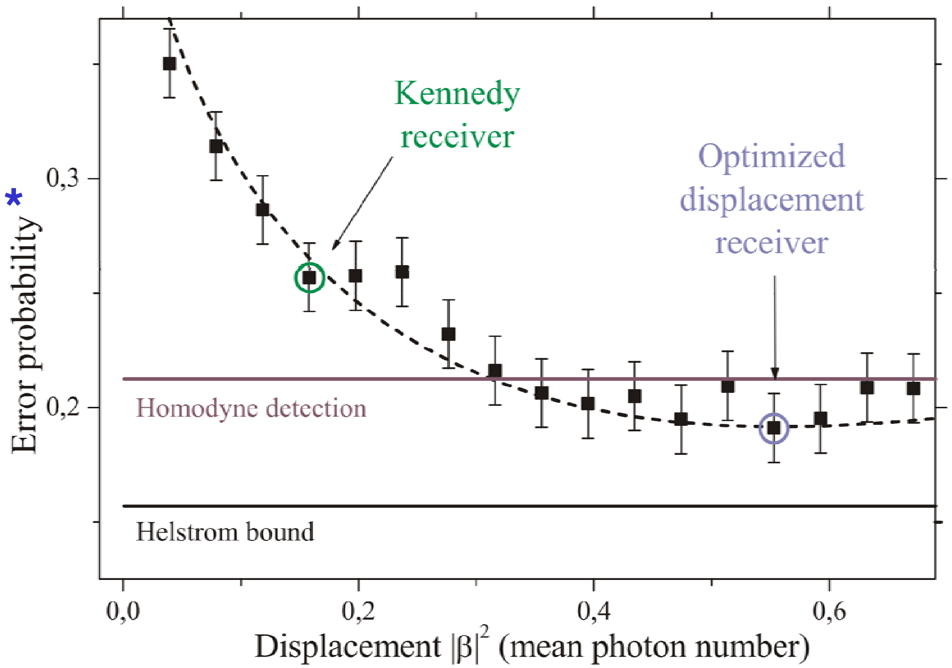
Optimal Displacement Receiver

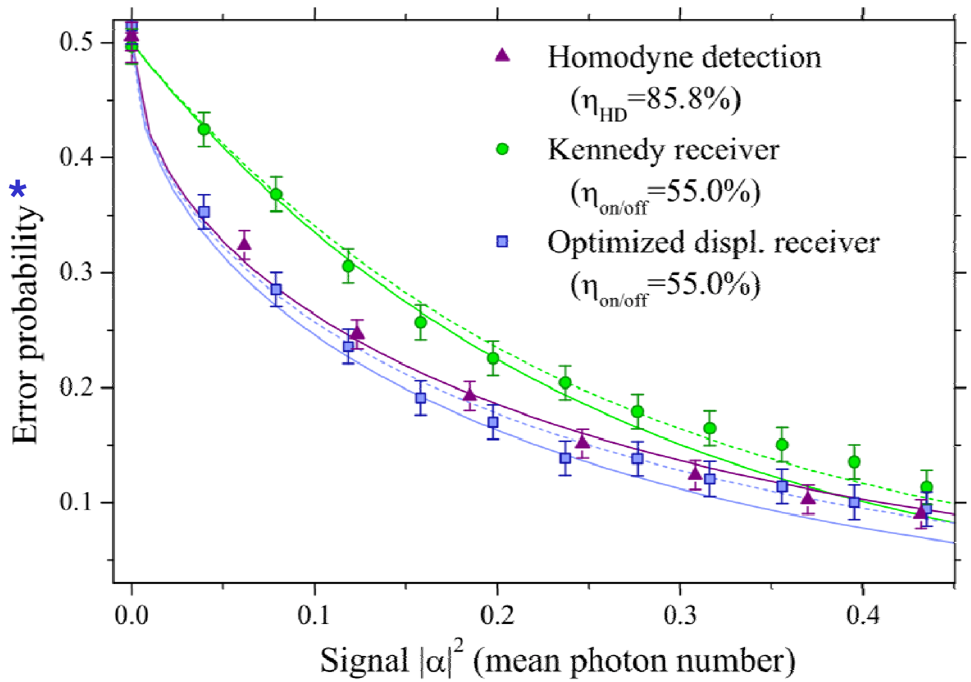
Wittmann, et al., Phys. Rev. Lett. 101, 210501 (2008)





Average error probability (experimental)





***Detection efficiency compensated**

⇨ **"Proof-of-principle" demonstration succeeded!**

Wittmann, et al., Phys. Rev. Lett. 101, 210501 (2008)

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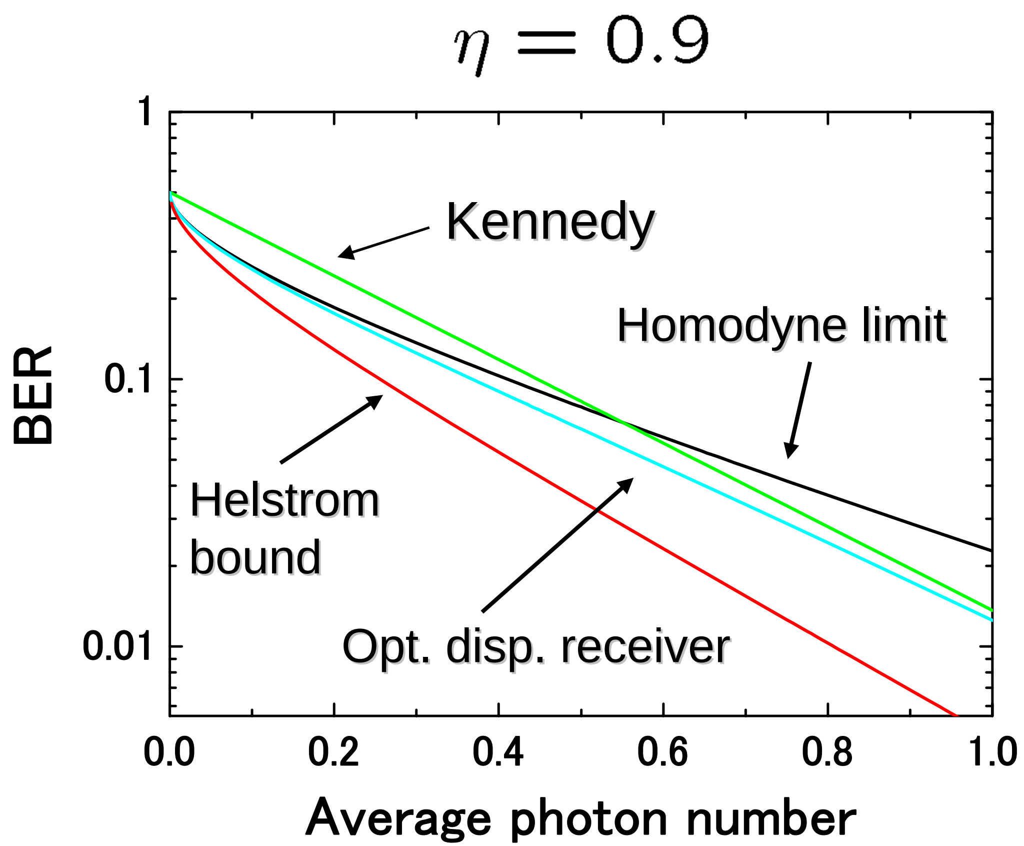
Detector requirements



to beat
the homodyne limit…

Detector

QE > 90%

DC < 10^{-3}

Visibility

ξ > 0.995

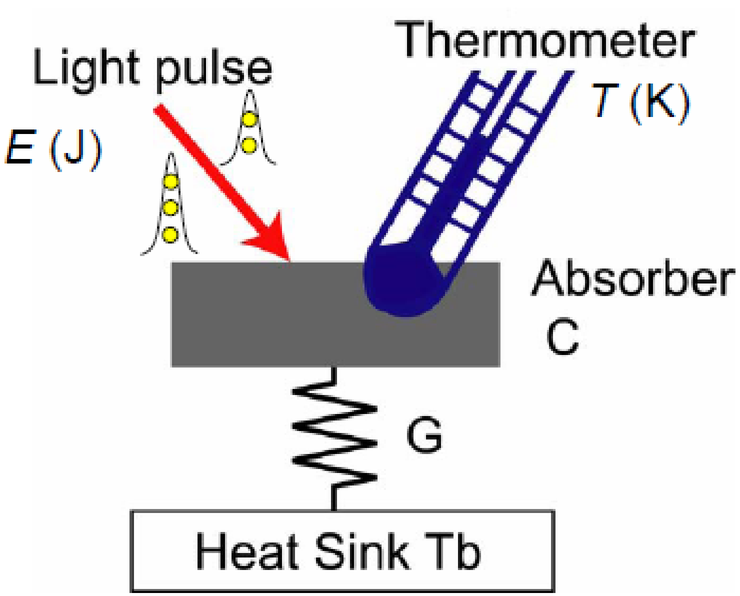
Advanced detectors?

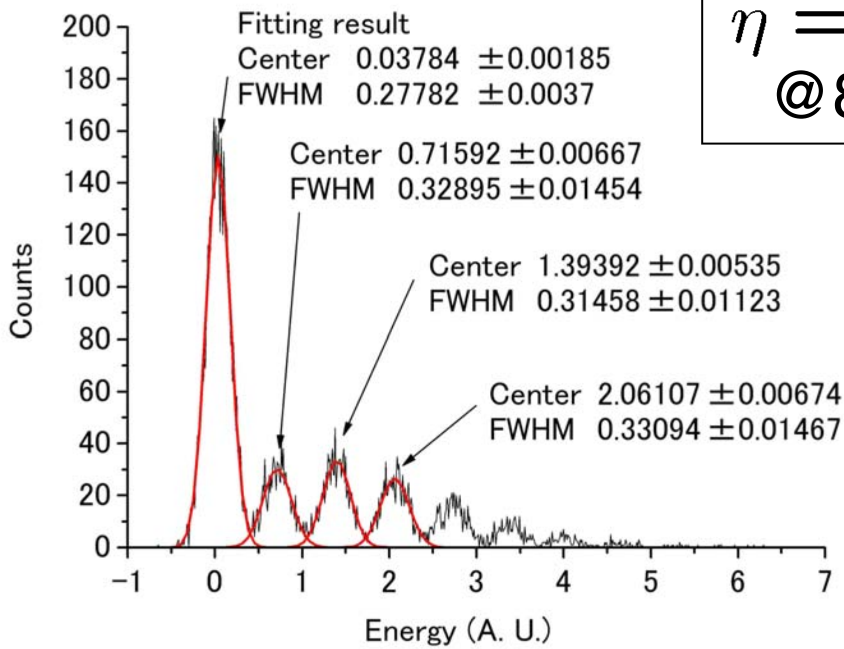
Transition Edge Sensor (TES)

TES: calorimetric detection of photons

Fukuda et al., (2009) @AIST

Schematics of calorimeters





$\eta = 0.92$
@850nm

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Cut-off rate evaluation

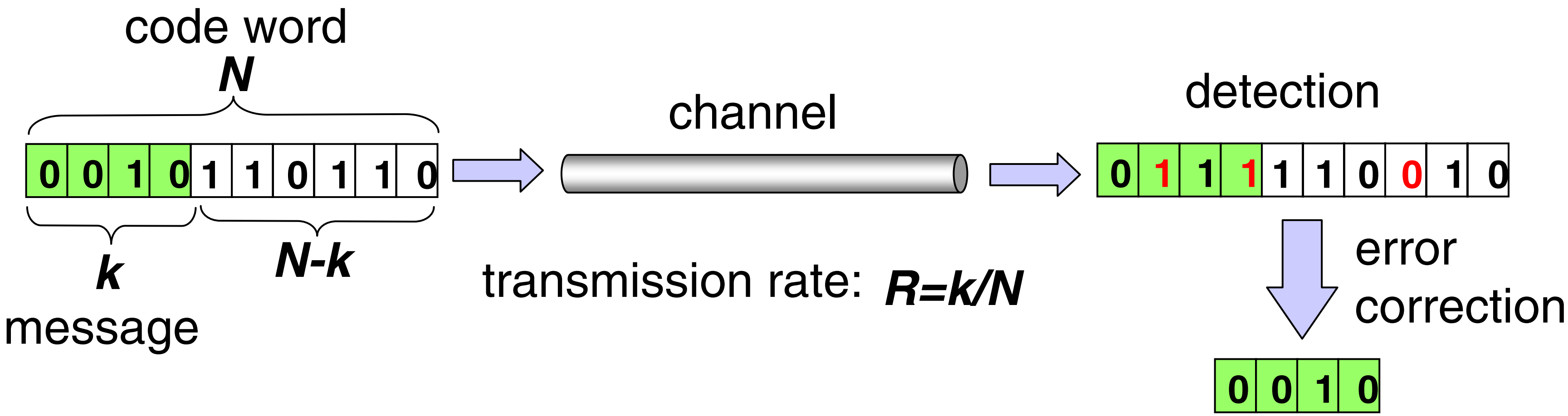
1. (Classical) reliability function and cut-off rate

2. Quantum measurement attaining the maximum cut-off rate

3. Receiver implementation & simulation

4. Conclusions

Reliability function

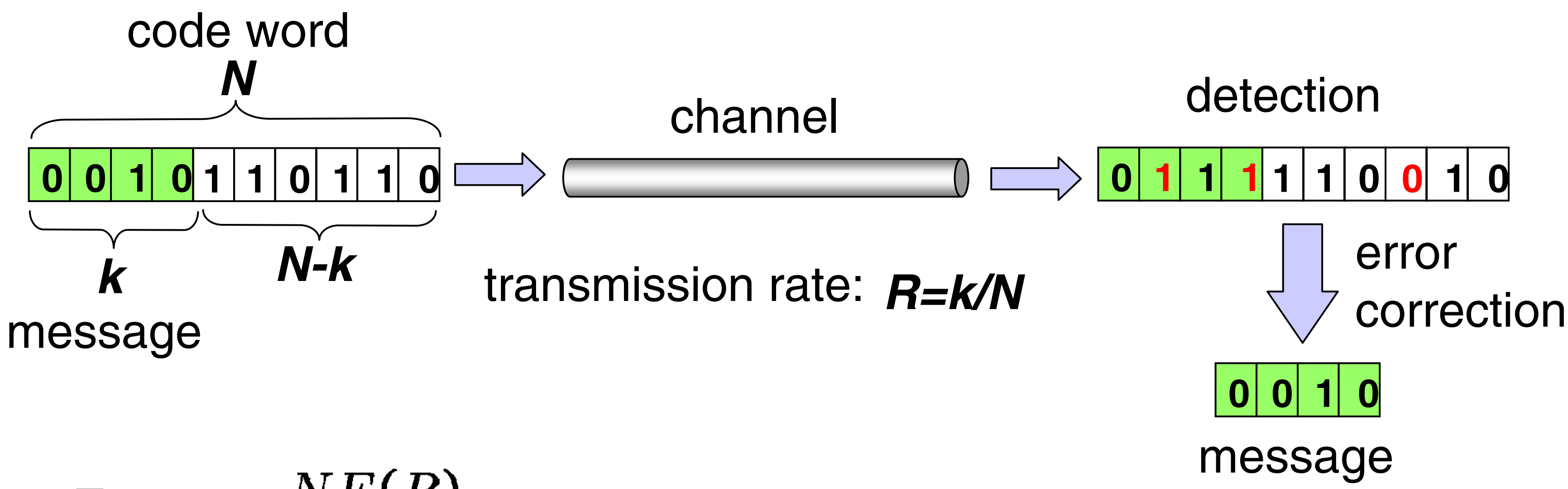


Reliability function $E(R)$ → Error bound for finite N $P_e \leq e^{-NE(R)}$

Average error probability (BER) P_{av} → Detection error (w/o coding)

Mutual information (Shannon information) $I(X:Y)$ → Maximum R preserving $P_e \to 0$ at $N \to \infty$

Reliability function and cut-off rate



$$P_e \leq e^{-NE(R)}$$

Reliability function $E(R)$

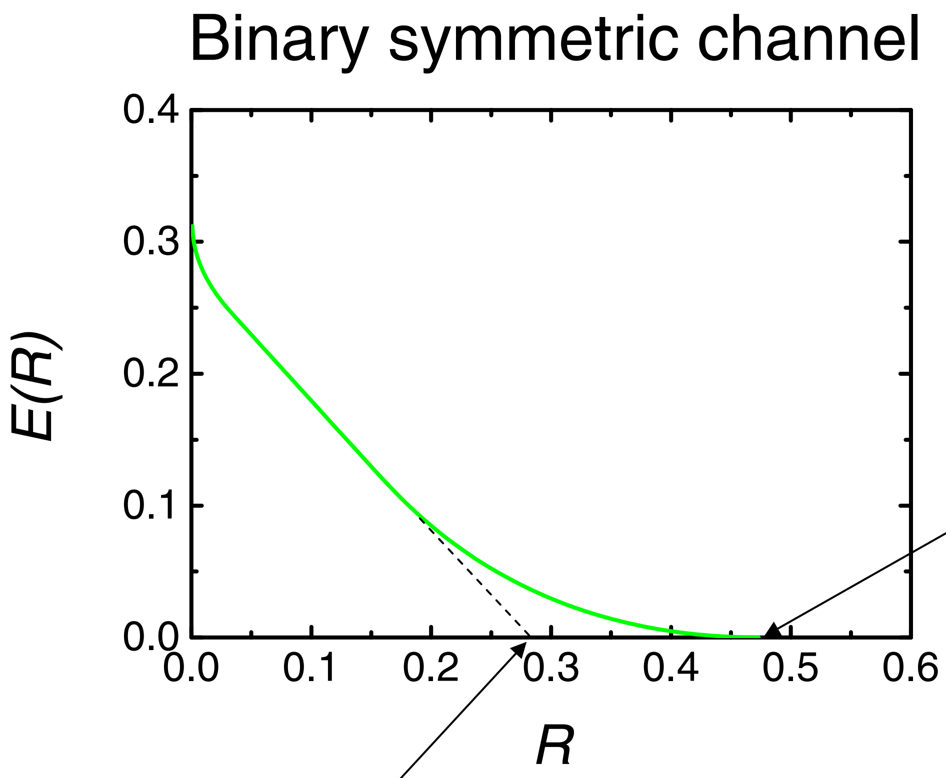
$$E(R) = \max_{\rho \in (0,1], \mathbf{p}} [-\rho R + E_0(\rho, \mathbf{p})]$$

$$E_0(\rho, \mathbf{p}) = -\ln \sum_{j=1}^{m} \left(\sum_{i=1}^{n} p_i P(j|i)^{1/(1+\rho)} \right)^{1+\rho}$$

Gallager, *Information Theory and Reliable Communications*, (1968).

Reliability function and cut-off rate (classical)

$$P_e \leq e^{-NE(R)}$$



$I(X:Y)$

: mutual information

R_c : cut-off rate

$$R_c(\mathbf{p}) = -\ln \min_{\mathbf{p}} \sum_{j=1}^{m} \left(\sum_{i=1}^{n} p_i \sqrt{P(j|i)} \right)^2$$

Binary communication

sender

$|\text{-}\alpha\rangle, |\alpha\rangle$

BPSK coherent signal

receiver

homodyne or
quantum receivers

- fixed single-shot measurement
  (non-adaptive, not collective)

Cut-off rate upper bound

$$R_c \leq \ln\left(\frac{2}{1+\kappa}\right)$$

$$\kappa = |\langle\alpha|\text{-}\alpha\rangle|$$

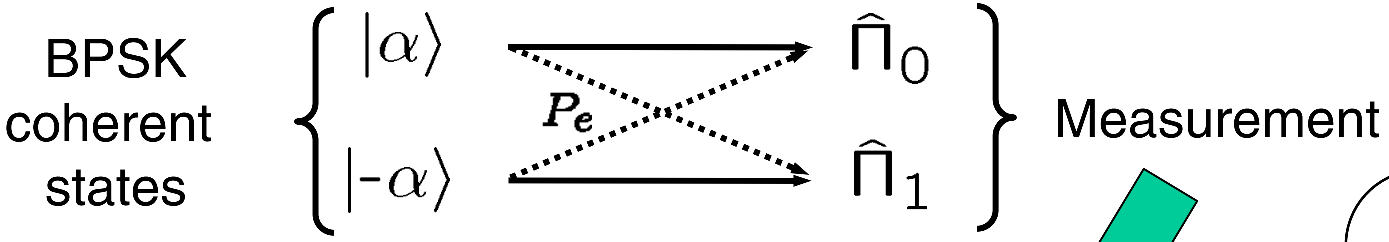
Bendjaballah and Charbit, IEEE Trans. Info. Theory, 35, 1114 (1989).

Optimal quantum measurement strategies

We found that the following three strategies simultaneously attaining the upper bound of the cut-off rate;

- **Minimum (average) error discrimination**
- **Unanimous voting discrimination**
- **Unambiguous state discrimination**

Minimum (average) error discrimination



$$\hat{\Pi}_i = |\pi_i\rangle\langle\pi_i| \quad \begin{cases} |\pi_0\rangle = a|\alpha\rangle - b|-\alpha\rangle \\ |\pi_1\rangle = b|\alpha\rangle - a|-\alpha\rangle \end{cases}$$

$$a = \sqrt{\frac{1+\sqrt{1-\kappa^2}}{2(1-\kappa^2)}}$$

$$b = \sqrt{\frac{1-\sqrt{1-\kappa^2}}{2(1-\kappa^2)}}$$

$$\kappa = |\langle\alpha|-\alpha\rangle|$$

→ Projection onto the superpositions of coherent states

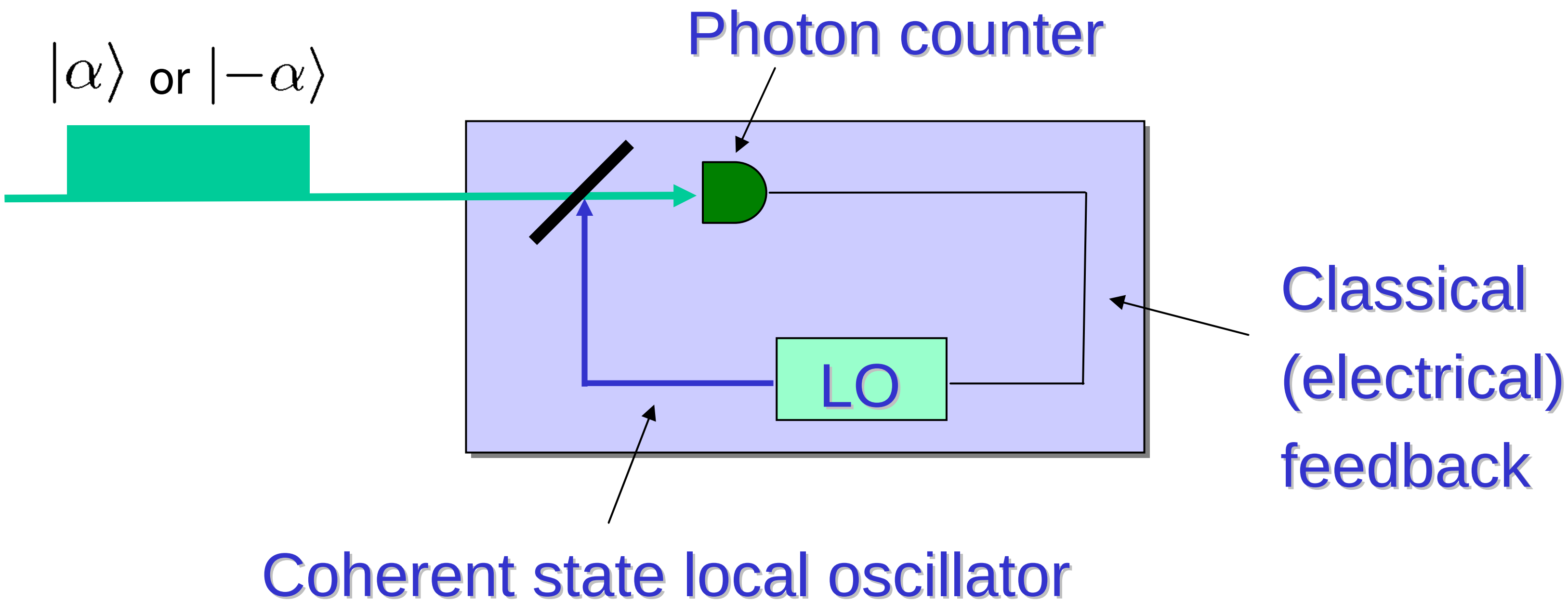
Minimum Error Probability: $P_e^{ME} = \frac{1}{2}\left(1-\sqrt{1-\kappa^2}\right)$

I(X:Y) is also maximized.

Cut-off rate: $R_c = \ln\left(\frac{2}{1+\kappa}\right)$

Implementation: realtime adaptive feedback Dolinar receiver

S. J. Dolinar, RLE, MIT, QPR, 111, 115, (1973)

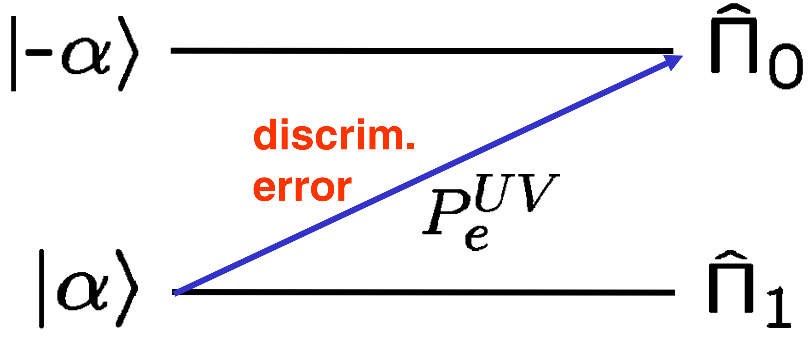


Concept demonstration

Cook, Martin, and Geremia, Nature 446, 774 (2007)

Difficult to implement with high visibility & high QE detectors?

Unanimous voting discrimination



$$\hat{\Pi}_0 = |{-\alpha}\rangle\langle{-\alpha}|$$
$$\hat{\Pi}_1 = \hat{I} - |{-\alpha}\rangle\langle{-\alpha}|$$

$$P_e^{UV}/2 > P_e^{ME}$$

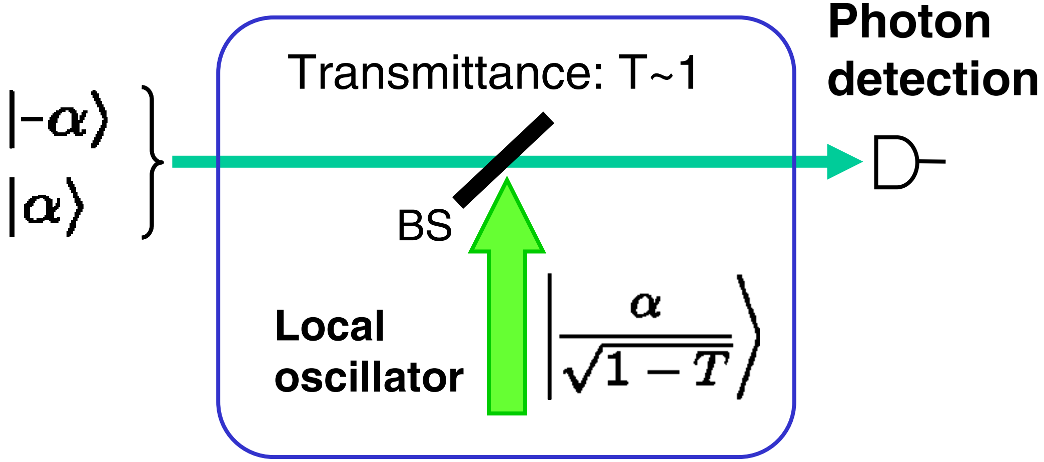
Cut-off rate: max.!

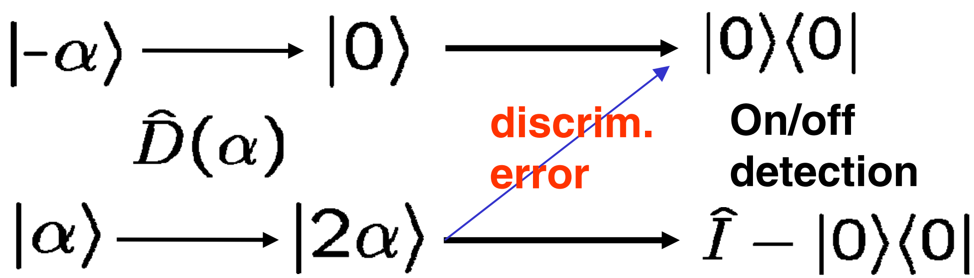
$$R_c = \ln\left(\frac{2}{1+\kappa}\right)$$

$$\kappa = |\langle\alpha|{-\alpha}\rangle|$$

Kennedy receiver

Displacement operation

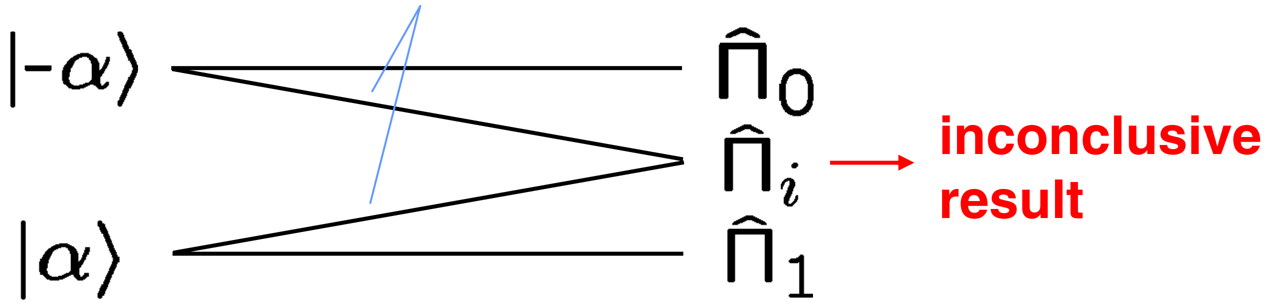




Unambiguous state discrimination

Ivanovic, Phys. Lett. A 123, 257 (1987)
Dieks, Phys. Lett. A 126, 303 (1988)
Peres, Phys. Lett. A 128, 19 (1988)

$$P_i = \kappa = |\langle \alpha | \text{-}\alpha \rangle|$$



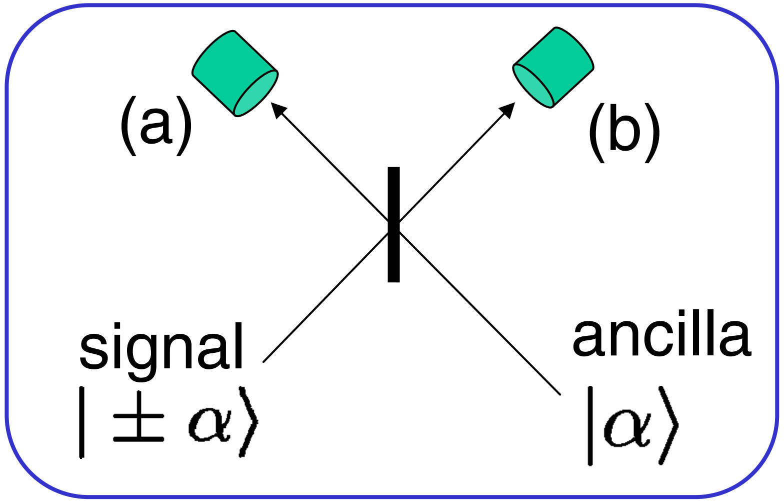
$$P_i > P_e^{ME}$$

Cut-off rate: ⇨ Maximum!

$$R_c = \ln\left(\frac{2}{1+\kappa}\right)$$

Implementation of USD

van Enk, Phys. Rev. A 66, 042313 (2002).



Signal decision

(a) click, (b) no ⇨ $|\alpha\rangle$

(a) no, (b) click ⇨ $|-\alpha\rangle$

(a) no, (b) no ⇨ **inconclusive**

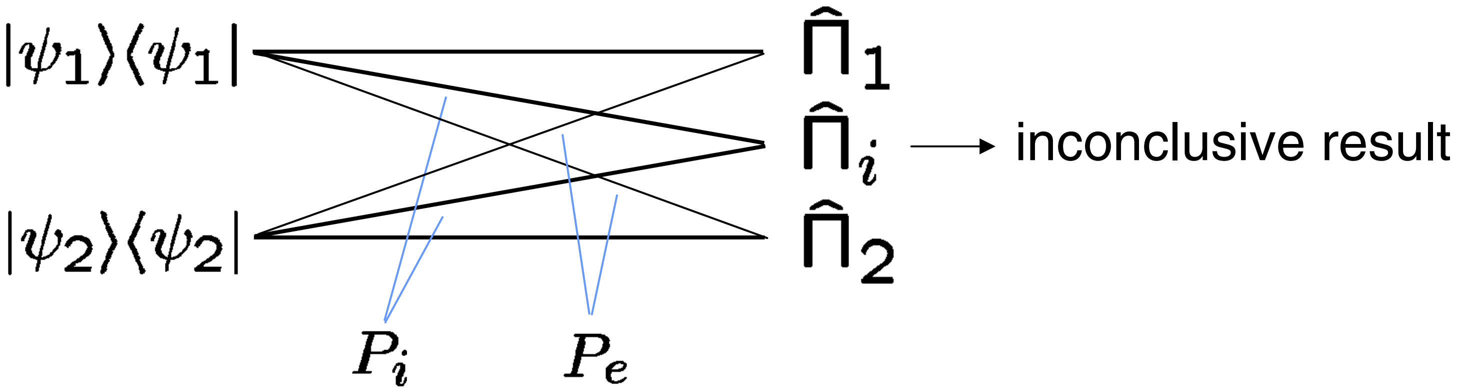
in practice,

(a) click, (b) click ⇨ **inconclusive**

Optimal intermediate measurement

Intermediate between
unambiguous & min. error discrimination

Chefles and Barnett, J. Mod. Opt. 45, 1295 (1998).



$$P_i(1 - P_i - P_e) \geq \frac{1}{4}(\kappa - P_i)^2$$

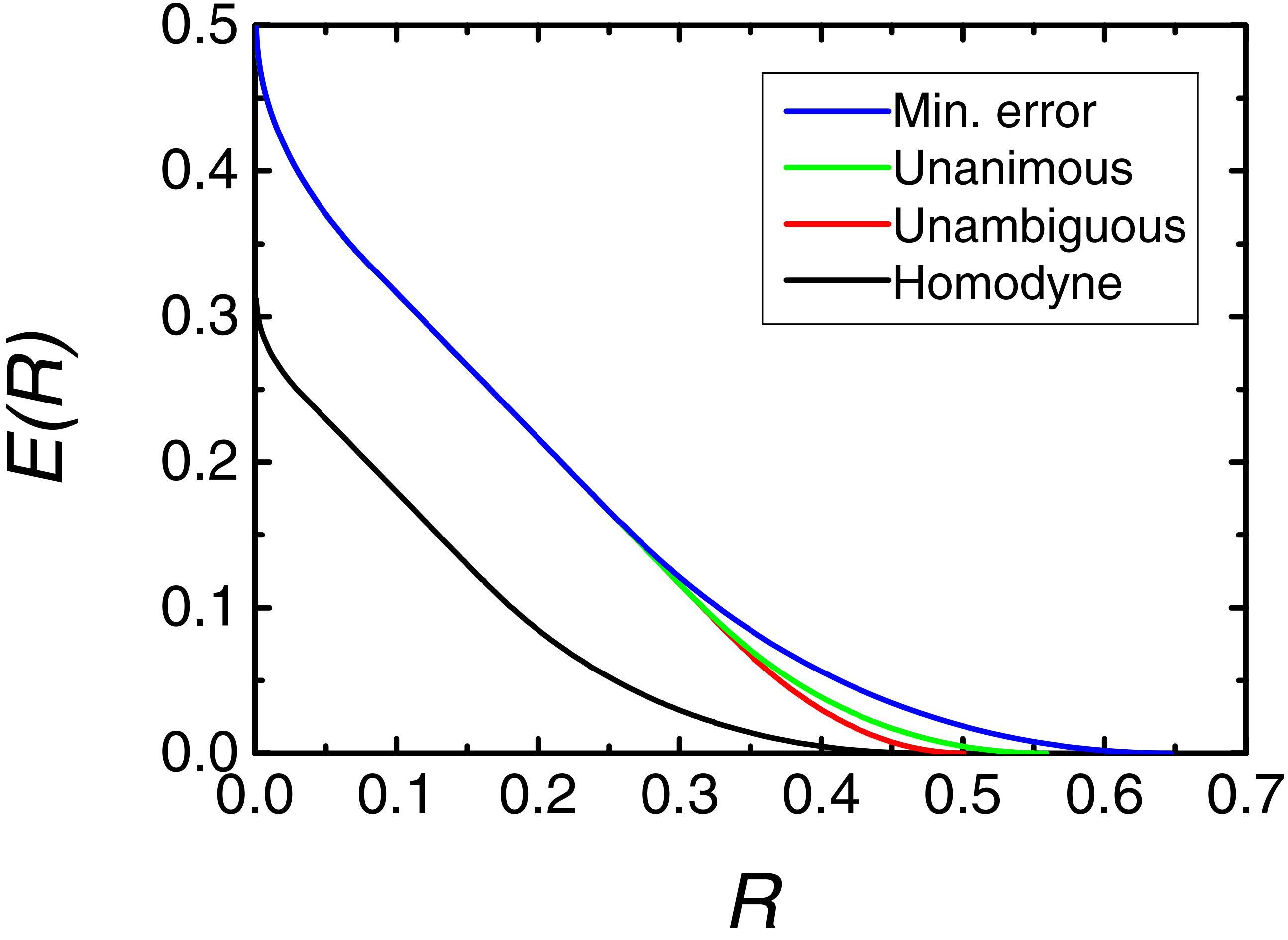
$\kappa = |\langle\psi_1|\psi_2\rangle|$ $\quad 0 \leq P_i \leq \kappa$

$0 \leq P_e \leq (1 - \sqrt{1 - 4\kappa^2})/2$

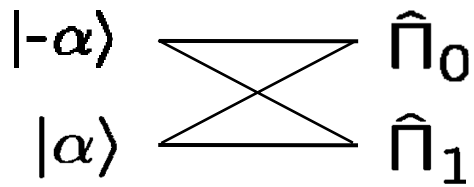
$$\Rightarrow \quad R_c = \ln\left(\frac{2}{1+\kappa}\right)$$

Reliability functions (& cut-off rate)

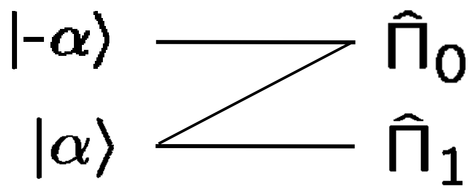
BPSK coherent signal $\langle -\alpha | \alpha \rangle = 0.5$



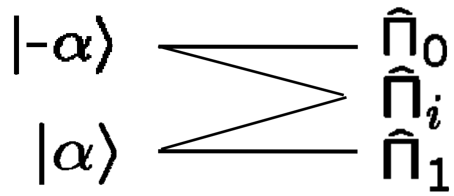
Minimum Error



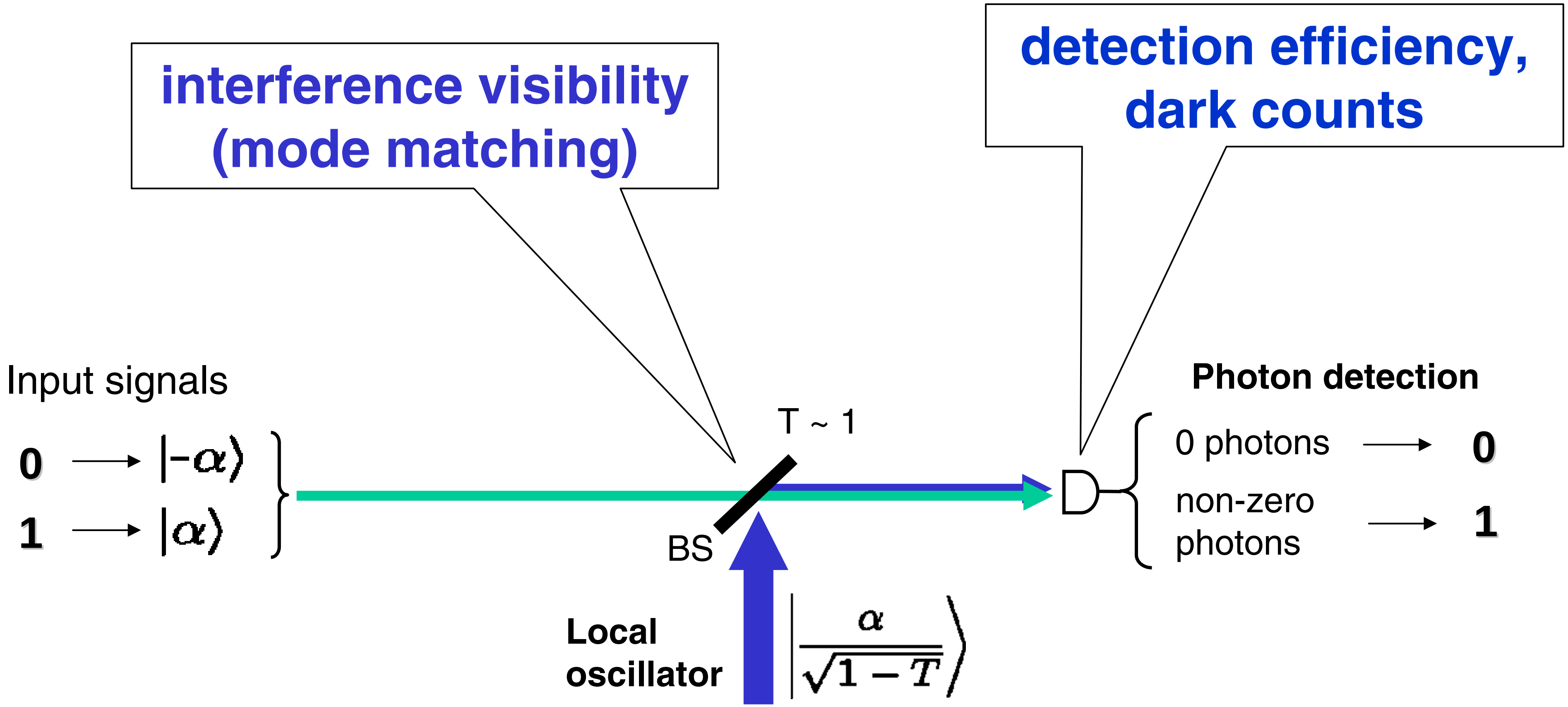
Unanimous Voting



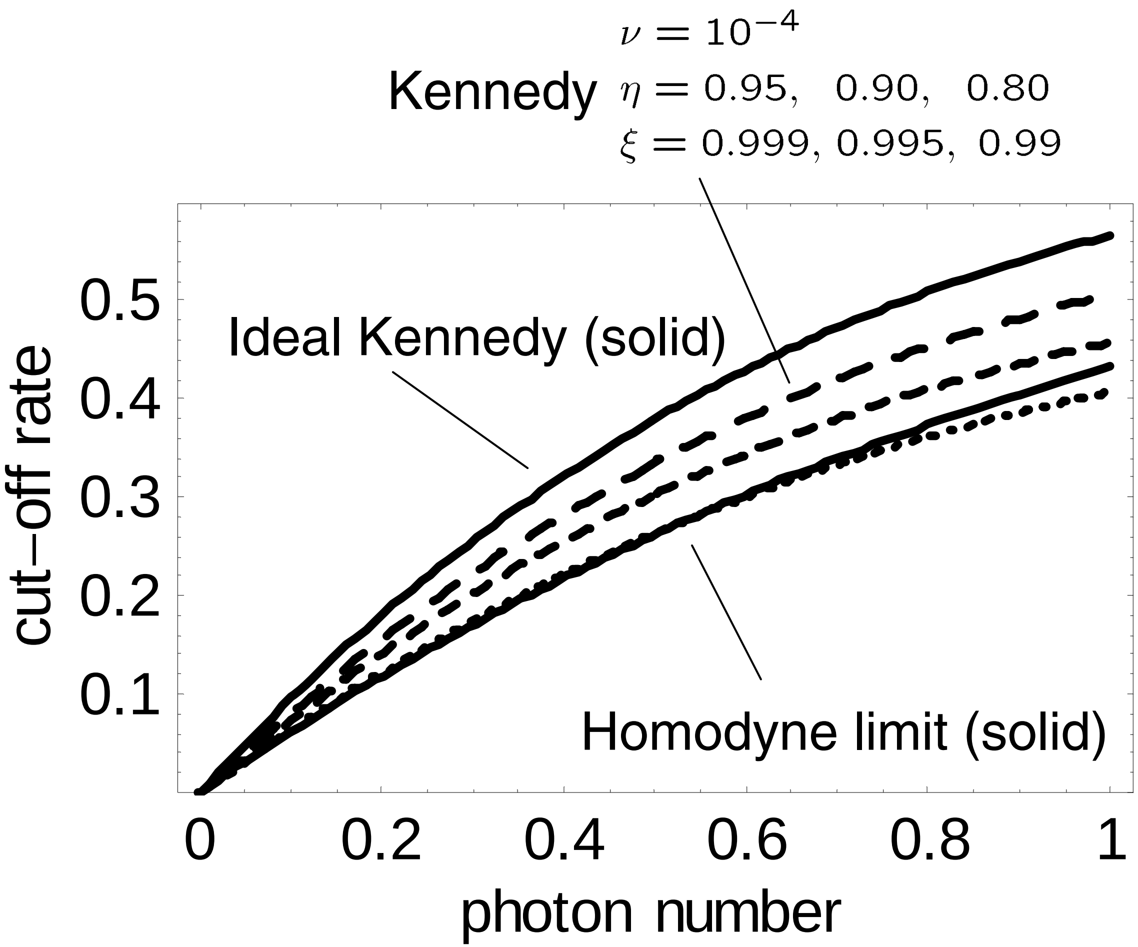
Unambiguous



Against the imperfections…



Cut-off rate performance (Kennedy receiver)



$T = 0.99$

ξ : mode match (visibility)

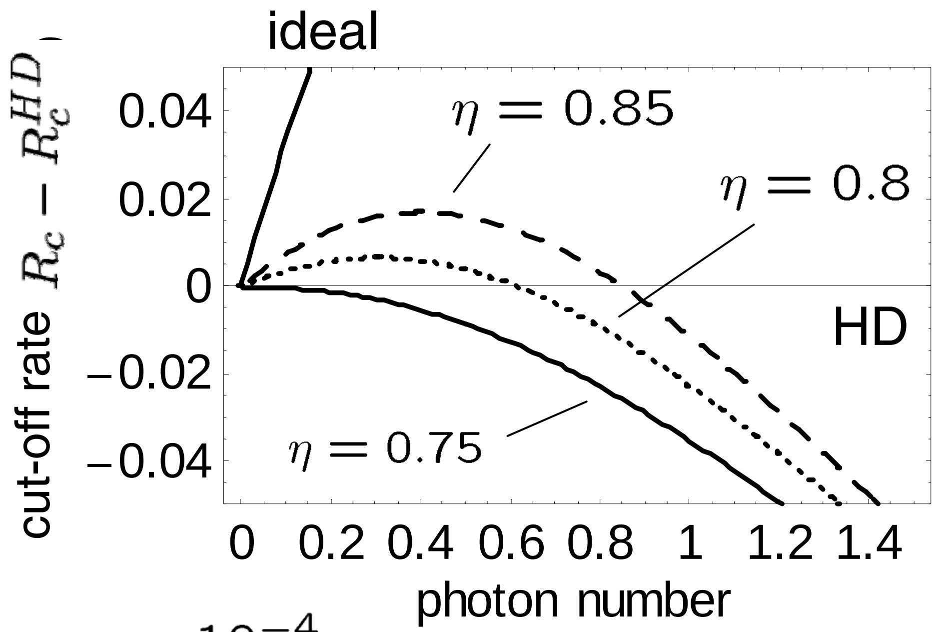
η : quantum efficiency

ν : dark counts

Cut-off rate performance

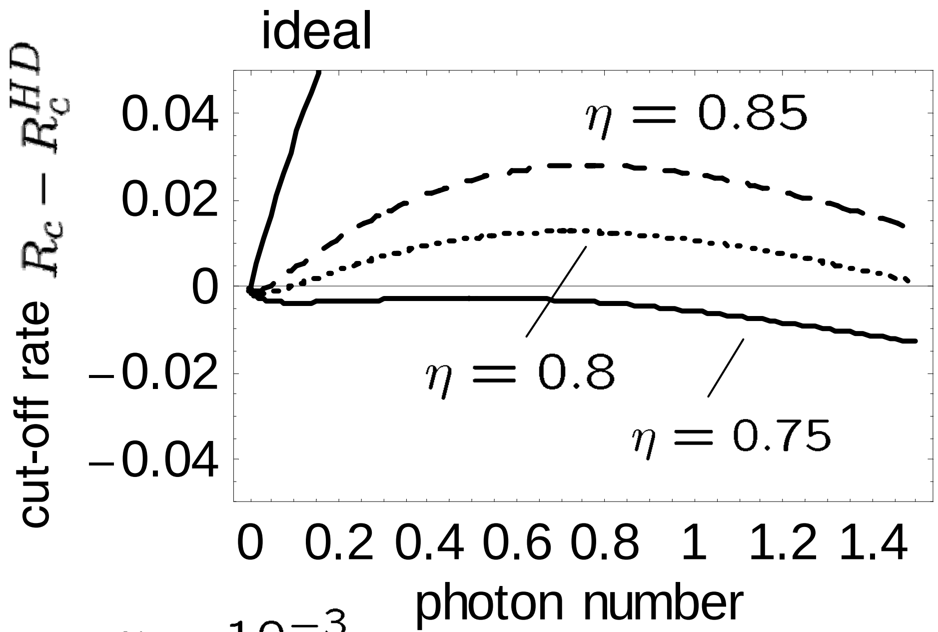
Difference: $R_c - R_c^{HD}$

Kennedy receiver



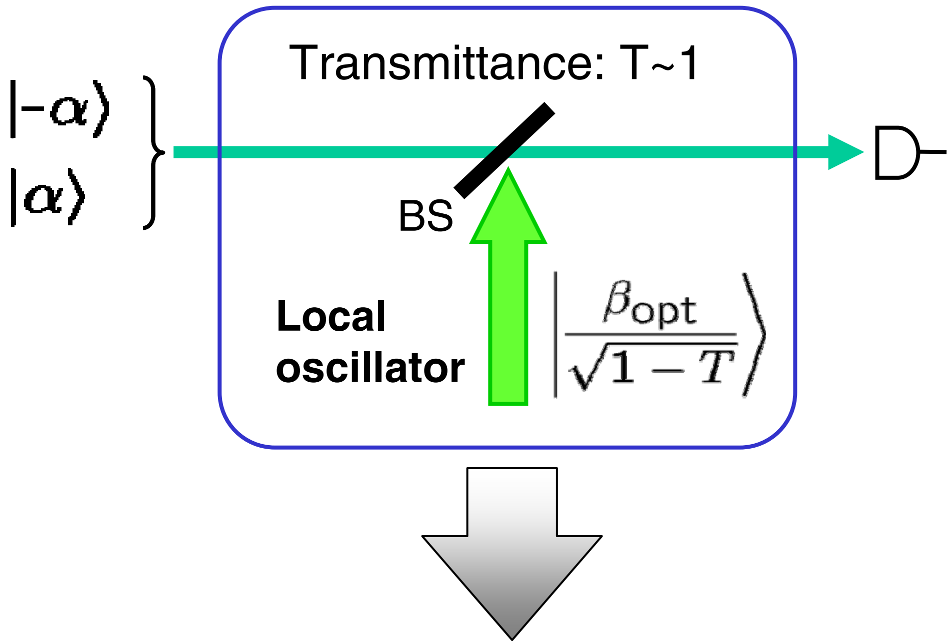
$\nu = 10^{-4}$
$\xi = 0.99$

USD receiver



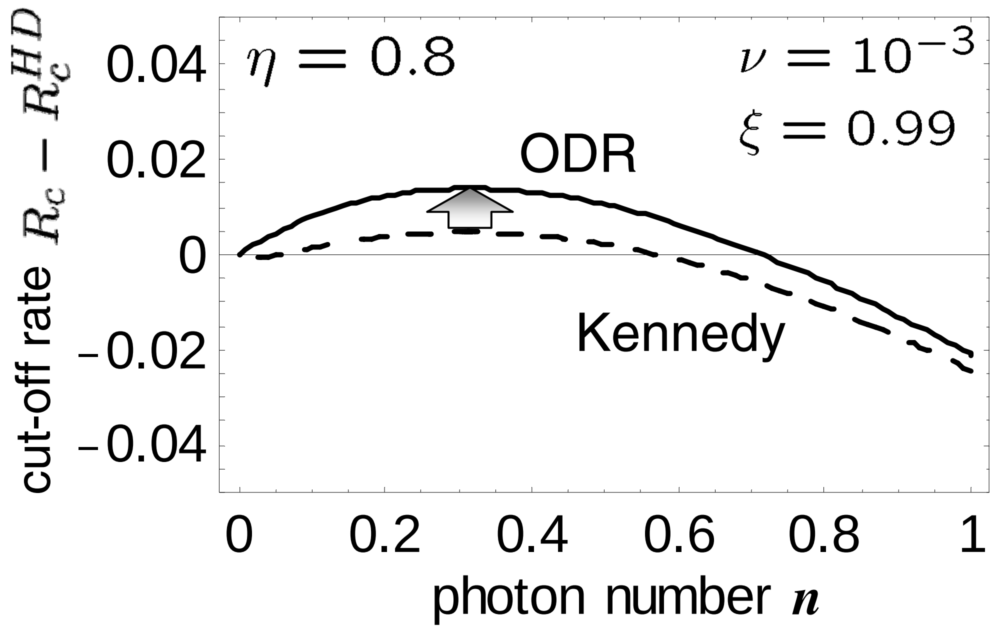
$\nu = 10^{-3}$
$\xi = 0.99$

Optimal displacement receiver



$$\hat{D}(\alpha) \rightarrow \hat{D}(\beta_{opt})$$

Optimization taking into account practical imperfections



Comparable to the USD receiver for $n < 0.5$

Easier to implement!

Optimal displacement receiver with a TES

would be the first experimental demonstration beating the homodyne limit

Under construction…

Conclusions

1. Homodyne measurement is the optimal GOCC measurement for the minimum error discrimination of binary coherent states.

⇨ State discrimination via Gaussian operations and classical communication

2. Near-optimal quantum receiver beyond the homodyne limit

Figure of merits: - min. error probability
- reliability function & cut-off rate

⇨ Simplest and robust scheme

⇨ Optimal displacement measurement

Proof-of-principle experiment

Experiment beyond a "proof-of-principle" …

Detector

QE > 90%

DC < 10^{-3}

Mode match

$\xi > 0.995$

⇩

Detector

QE > 80%

DC < 10^{-3}

Mode match

$\xi > 0.990$