# Quantum Benchmarks for Gaussian States

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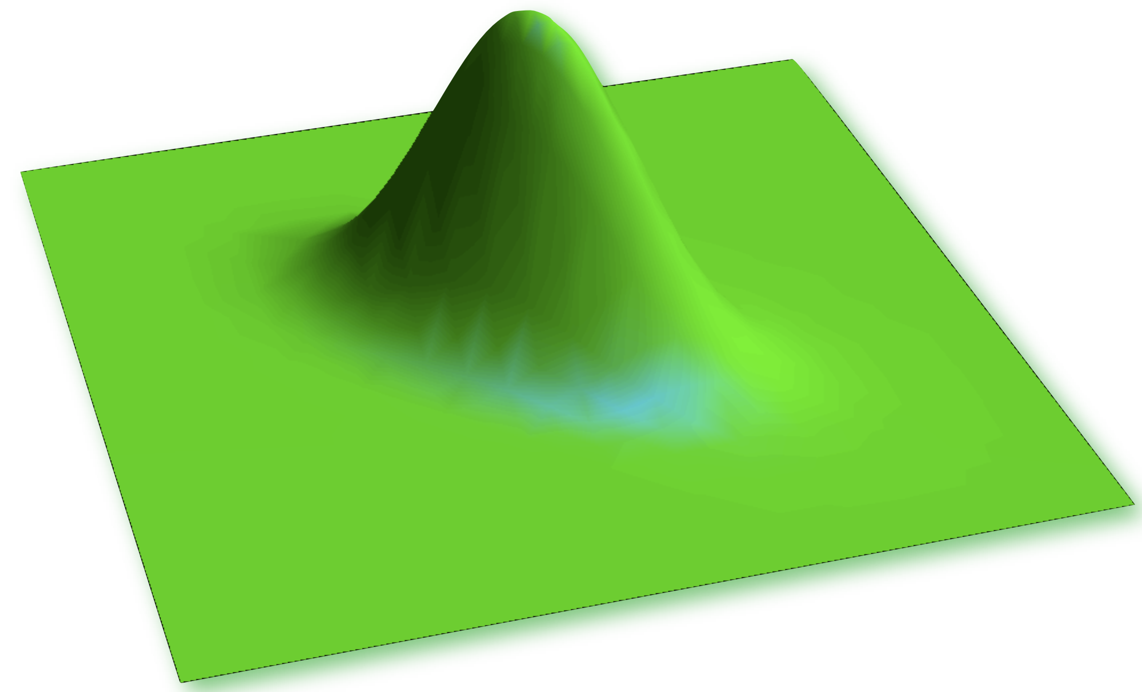
# Gaussian States

- A family of continuos variable quantum states defined by a Gaussian characteristic function.



- Canonical form:

$$\rho = D(\alpha)S(r,\phi)\rho_\beta S(r,\phi)^\dagger D(\alpha)^\dagger$$

$$\rho_\beta = \frac{\mathrm{e}^{-\beta\hat{n}}}{\mathrm{tr}\left(\mathrm{e}^{-\beta\hat{n}}\right)} \qquad D(\alpha) = \mathrm{e}^{\alpha a^\dagger - \alpha^* a} \quad S(r,\phi) = \exp\left[r/2(a^2\mathrm{e}^{-i2\phi} - a^{\dagger 2}\mathrm{e}^{i2\phi})\right]$$

- Very good description of states of light produced in labs (laser produces coherent state + passive/active optical operations)

# Quantum Benchmarks

**IDEAL**

*identity channel*

$$\rho \longrightarrow \boxed{\mathcal{E}(\rho)} \longrightarrow \rho$$

e.g. quantum teleportation
or quantum memories

**REAL**

*noisy identity channel*

$$\rho \longrightarrow \boxed{\tilde{\mathcal{E}}(\rho)} \longrightarrow \tilde{\rho}$$

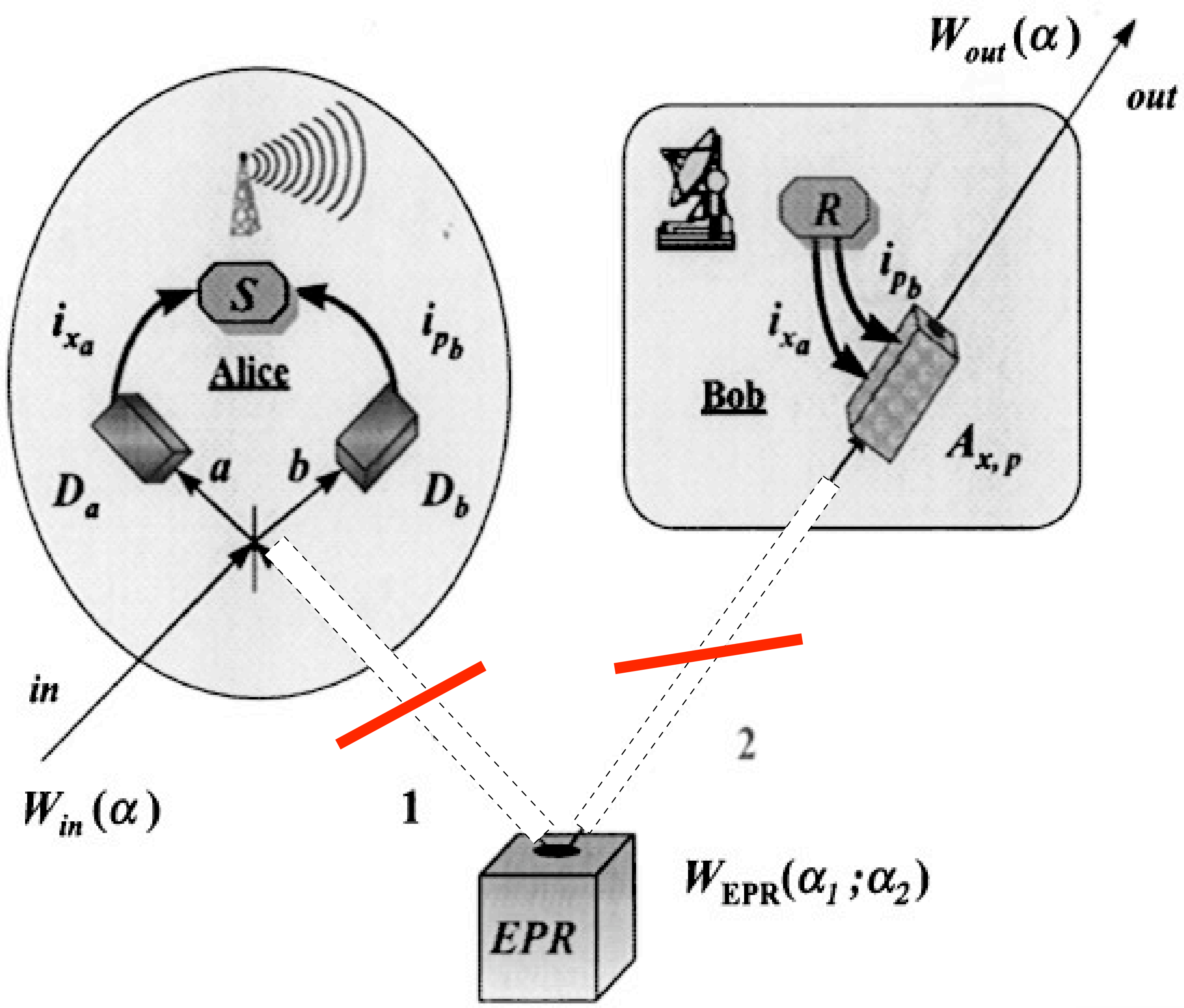
?
e.g. quantum teleportation
or quantum memories

Are quantum resources necessary to emulate the channel?

# Are quantum resources necessary to emulate the channel?

First threshold for quantum teleportation of coherent states:

- Fidelity of output state when no quantum correlations are used.




$$\mathcal{F}_{\text{tel}} = \langle\alpha|\,\rho_{\text{out}}\,|\alpha\rangle =$$

$$= \langle\alpha| \left( \int d\beta^2 \text{tr}(E_\beta\,|\alpha\rangle\langle\alpha|)\,|\beta\rangle\langle\beta| \right) |\alpha\rangle =$$

$$= \frac{1}{\pi}\int d^2\beta |\langle\alpha|\beta\rangle|^4 = \frac{1}{\pi}\int d^2\beta \mathrm{e}^{-2|\alpha-\beta|^2} = \frac{1}{2}$$

$$\text{where } E_\beta = \frac{1}{\pi}\,|\beta\rangle\langle\beta|$$

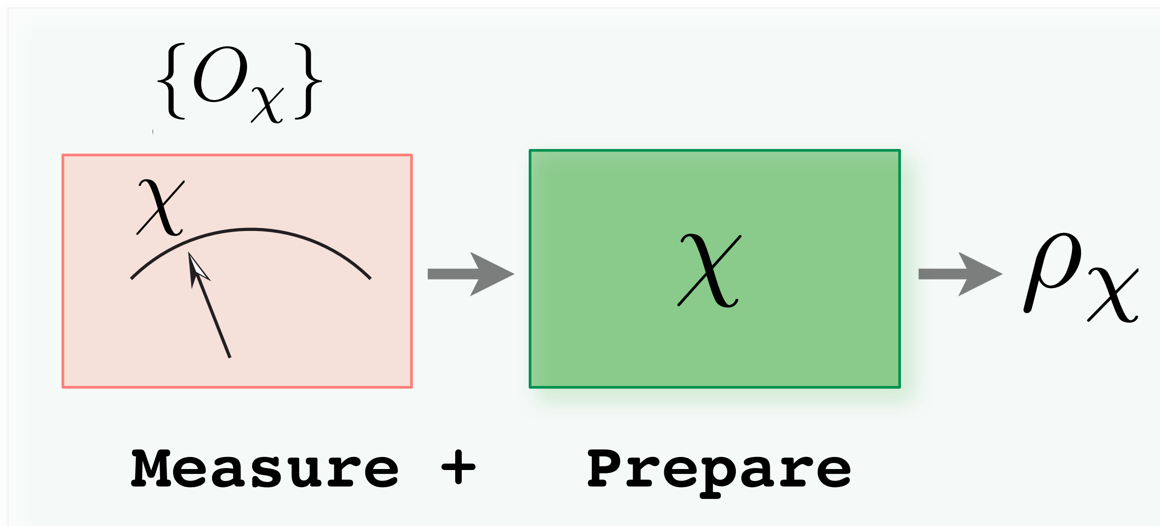
$$\mathcal{F} = \langle\alpha|\rho_{\text{out}}|\alpha\rangle > 1/2.$$

→ Quantum resources are being used.

A. Furusawa et. al. Science 1998

## More rigorous quantum benchmark (Braunstein, Fuchs & Kimble JMO 2000):

$\rho_{\text{in}} \in \Omega \longrightarrow$



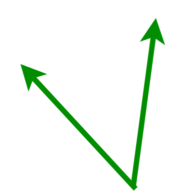

$\rightarrow \rho_\chi \longrightarrow$

$$\rho_{\text{out}} = \sum_\chi \text{tr}(\rho_{\text{in}} O_\chi)\rho_\chi$$

$$\mathcal{F} = \int \langle\psi_{\text{in}}|\, \rho_{\text{out}}\, |\psi_{\text{in}}\rangle\, P(|\psi_{\text{in}}\rangle) d\, |\psi_{\text{in}}\rangle$$

**Different choices of input-state families:**

- 2 non-orthogonal states: $|\psi_0\rangle, |\psi_1\rangle$



$x = \langle\psi_0|\psi_1\rangle$

$$\mathcal{F} = \frac{1}{2}\left(1 + \sqrt{1 - x^2 + x^4}\right) \geq 0.933$$

- Isotropic distribution (all pure states with equal probability): $\mathcal{F} = \frac{2}{d+1} \xrightarrow{d\to\infty} 0$

- Coherent states with Gaussian distribution of amplitudes: (Braunstein, et al.)

$$p(\alpha) = \frac{\lambda}{\pi} e^{-\lambda|\alpha|^2} \qquad \mathcal{F}_{coh} = \frac{1+\lambda}{2+\lambda} \xrightarrow{\lambda\to 0} 1/2$$

Hammerer et. al. PRL 2005

- Micro-canonical ensemble of pure Gaussian states (Serafini et al PRL 2007)

We consider Gaussian phase-covariant family of input states:

$$\rho_{\mathrm{in}}^{\phi} = U(\phi)\rho_0 U(\phi)^{\dagger} \qquad \phi \in [0, 2\pi)$$

where $\rho_0$ is a Gaussian state (pure or mixed)

and $U(\phi) = \mathrm{e}^{i\phi a^{\dagger} a}$

- **Benchmark**

$$\mathcal{F}_{\mathrm{cl}} = \int \frac{d\phi}{2\pi} F(\rho_{\mathrm{in}}^{\phi}, \rho_{\mathrm{av}}^{\phi})$$

$$\begin{cases} F(\rho_1, \rho_2) = (\mathrm{tr}|\sqrt{\rho_1}\sqrt{\rho_2}|)^2 \\ \rho_{\mathrm{av}}^{\phi} = \sum_{\chi} p(\chi|\rho_{\mathrm{in}}^{\phi})\rho_{\chi} \end{cases}$$

- Phase-covariant family is large enough to give reasonable benchmarks, and is easy to produce experimentally.

- Adesso & Chiribella [PRL 2008] have also studied benchmarks with mixed states taking as input family: $\rho(r) = \hat{S}(r)\rho_\beta \hat{S}(r)^\dagger$

$$\mathcal{F}_{AC} = \sum_\chi \int dr P(r) p(\chi|r) F[\rho(r), \rho(r_\chi)] \leq$$

$$\leq \int dr P(r) F[\rho(r), \sum_\chi p(\chi|r)\rho(r_\chi)] = \mathcal{F}_{\mathrm{cl}}$$

fidelity is concave

  - Optimal Guess typically does not belong to family of input states.
  - Difficult to change squeezing parameter experimentally.

✳ Recently: phase covariant and displaced squeezed states (Owari et al.)

- Covariant strategies

Given a strategy $\{O_\chi = |\xi_\chi\rangle\langle\xi_\chi|\,, \rho_\chi\}$ one can define a *phase-shifted strategy* by

$O_{\chi,\theta} = U_\theta O_\chi U_\theta^\dagger,\ \rho_{\chi,\theta} = U_\theta \rho_\chi U_\theta^\dagger$ with at least the same fidelity:

$$\begin{aligned}\mathcal{F}^\theta &= \frac{1}{2\pi}\int d\phi F(\rho^\phi, \rho_{\text{av}}^{\phi,\theta}) = \\ &= \int d\phi \left( \text{tr} \left| U_\phi \sqrt{\rho_0} U_\phi^\dagger U_\theta \sqrt{\sum_\chi \text{tr}(U_\theta[\xi_\chi]U_\theta^\dagger U_\phi \rho_0 U_\phi^\dagger)\rho_\chi U_\theta^\dagger} \right| \right)^2 = \\ &= \int d\varphi F(\rho^\varphi, \rho_{\text{av}}^\varphi) = \mathcal{F}^{(\theta=0)}\end{aligned}$$

$\text{tr}|UBV| = \text{tr}|B|$

$\varphi = \phi - \theta$

where we have used the notation $[\psi] = |\psi\rangle\langle\psi|$

- Covariant strategies

Given set $\{O_\chi = |\xi_\chi\rangle\langle\xi_\chi|\,, \rho_\chi\}$ one can define a *covariant strategy* by

$O_{\chi,\theta} = 1/(2\pi) U_\theta O_\chi U_\theta^\dagger,\ \rho_{\chi,\theta} = U_\theta \rho_\chi U_\theta^\dagger$ with at least the same fidelity:

fidelity is concave

$$\mathcal{F}_{\rm cl} = \mathcal{F}^\theta = 1/2\pi \int d\theta \mathcal{F}^\theta \leq \int d\phi F\left(\rho^\phi, 1/2\pi \int d\theta \rho_{\rm av}^{\phi,\theta}\right) \equiv \mathcal{F}^{\rm cov}$$

$$\text{where } \rho_{\rm av}^{\phi,\theta} = \sum_\chi p(\chi_\theta|\rho^\phi)\rho_{\chi,\theta}$$

$$\mathcal{F}^{\rm cov} = \int d\phi \left( {\rm tr} \left| U_\phi \sqrt{\rho_0} U_\phi^\dagger \sqrt{\int \frac{d\theta}{2\pi} \sum_\chi {\rm tr}(U_\theta [\xi_\chi] U_\theta^\dagger U_\phi \rho_0 U_\phi^\dagger) U_\theta \rho_\chi U_\theta^\dagger} \right| \right)^2 =$$

$$= \left( {\rm tr} \left| \sqrt{\rho_0} \sqrt{\int \frac{d\varphi}{2\pi} \sum_\chi {\rm tr}(U_\varphi [\xi_\chi] U_\varphi^\dagger \rho^0) U_\varphi \rho_\chi U_\varphi^\dagger} \right| \right)^2 = \left( {\rm tr} \left| \sqrt{\rho^0} \sqrt{\rho_{\rm av}} \right| \right)^2$$

$$\text{where } \rho_{\rm av} = \int d\theta \sum_\chi p(\chi_\theta|\rho_0) \rho_{\chi,\theta}$$

The optimal classical fidelity (or quantum benchmark) can be conveniently written as,

$$\mathcal{F} = \left(\mathrm{tr}\left|\sqrt{\rho_0}\sqrt{\rho_{\mathrm{av}}}\right|\right)^2$$ with $$\rho_{\mathrm{av}} = \int d\theta \sum_{\chi} p(\chi,\theta|\rho_0)\rho_{\chi,\theta}$$

Note that for a single seed, the completeness relation fixes the POVM:

$$O_\theta = \frac{1}{2\pi} U_\theta[\xi] U_\theta^\dagger \quad \text{with} \quad |\xi\rangle = \sum_n |n\rangle$$ (up to some arbitrary phases)

The fidelity can be conveniently written as,

$$\mathcal{F} = \max_K \left(\mathrm{tr}_B \sqrt{\mathrm{tr}_A \sqrt{\rho_0} \otimes \sqrt{\rho_0} K \sqrt{\rho_0} \otimes \sqrt{\rho_0}}\right)^2$$

with $K = \int d\theta \sum_{\chi} O_{\chi,\theta} \otimes \rho_{\chi,\theta}$ $\qquad \rho_{\mathrm{av}} = \mathrm{tr}_A(\rho_0 \otimes \mathbb{1}\, K)$

i.e., $K \geq 0$, $\mathrm{tr}_B\, K = \mathbb{I}_A$, $U_\theta \otimes U_\theta$ invariant & separable.

- For pure states: $\rho_0 = |\psi_0\rangle\langle\psi_0|$ $\quad \mathcal{F} = \langle\psi_0|\langle\psi_0| K |\psi_0\rangle|\psi_0\rangle$ Navascues PRL 2008

Also, for fixed POVM with seeds $\{|\xi_\chi\rangle\langle\xi_\chi|\}$ the optimal fidelity can be written as,

$$\mathcal{F} = \sum_\chi \sup_{\psi_\chi} \langle\psi_\chi| A_\chi |\psi_\chi\rangle = \sum_\chi \|A_\chi\|_\infty \text{ with } A_\chi = \int d\phi/(2\pi) |\langle\xi_\chi|\psi_\phi\rangle|^2 |\psi_\phi\rangle\langle\psi_\phi|$$

$$A_\chi = \langle\xi_\chi|\Lambda|\xi_\chi\rangle \quad \text{with} \quad \Lambda = \int \frac{d\phi}{2\pi} |\psi_\phi\rangle|\psi_\phi\rangle\langle\psi_\phi|\langle\psi_\phi|$$

Hammerer PRL 2005

Optimal fidelity given by largest eigenvalue, and optimal guess given by corresponding eigenvector.

- If one restricts the guess-states to be in the input ensemble $\Omega$ , things also simplify considerably.

e.g. in the pure state case, the optimal POVM is known to be the single-seed POVM or canonical phase-measurement $|\xi\rangle = \sum_n |n\rangle$.

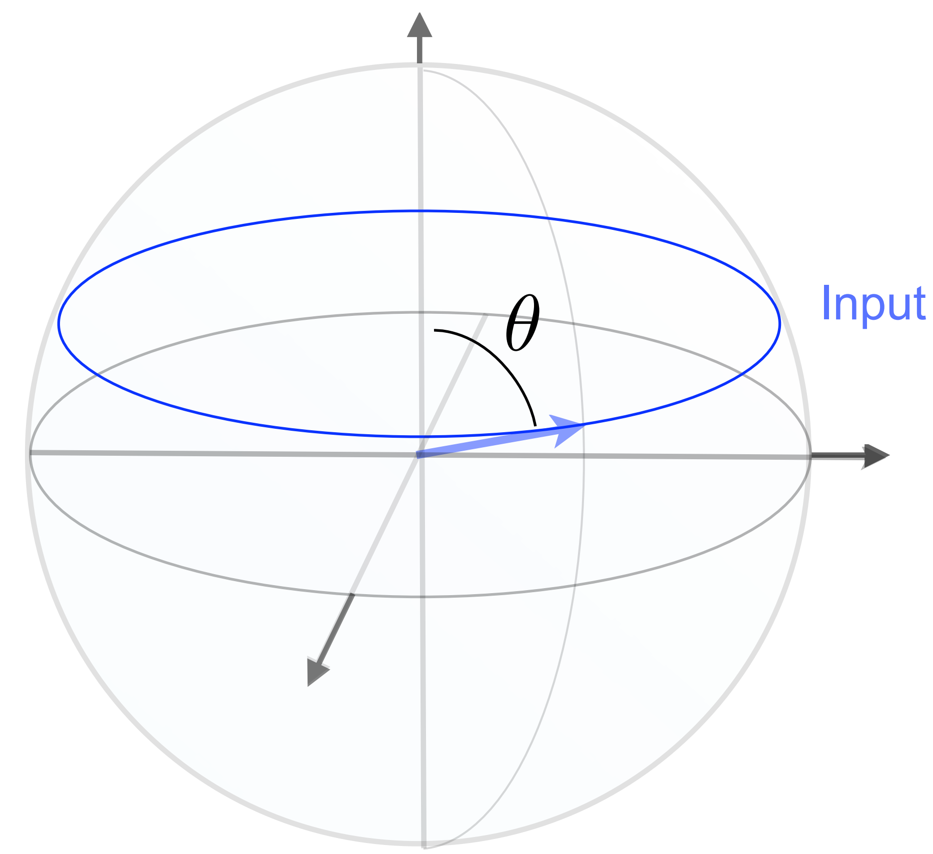
A. S. Holevo, Probab. & Stat. Aspects of Q. T., (1982)

- In general, no assumptions about the POVM nor the guess can be made, and we have to resort on numerical methods.

# Qubits

Can be solved with full generality.

$$\mathcal{F} = \max_K \left( \mathrm{tr}_B \sqrt{\mathrm{tr}_A \sqrt{\rho_0} \otimes \sqrt{\rho_0} K \sqrt{\rho_0} \otimes \sqrt{\rho_0}} \right)^2$$




$$\rho_0 = U_{y,\theta} \begin{pmatrix} \frac{1+r}{2} & 0 \\ 0 & \frac{1-r}{2} \end{pmatrix} U^\dagger_{y,\theta}$$

$$K = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1-a & b & 0 \\ 0 & b & 1-c & 0 \\ 0 & 0 & 0 & c \end{pmatrix}$$

$$\begin{cases} K \geq 0 \;\Leftrightarrow\; (1-a)(1-c) \geq |b|^2, \quad 0 \leq a \leq 1, \quad 0 \leq c \leq 1 \\ K^\Gamma \geq 0 \;\Leftrightarrow\; ac - |b|^2 \geq 0 \end{cases}$$

Change inequalities to equalities at the expense of additional variables,

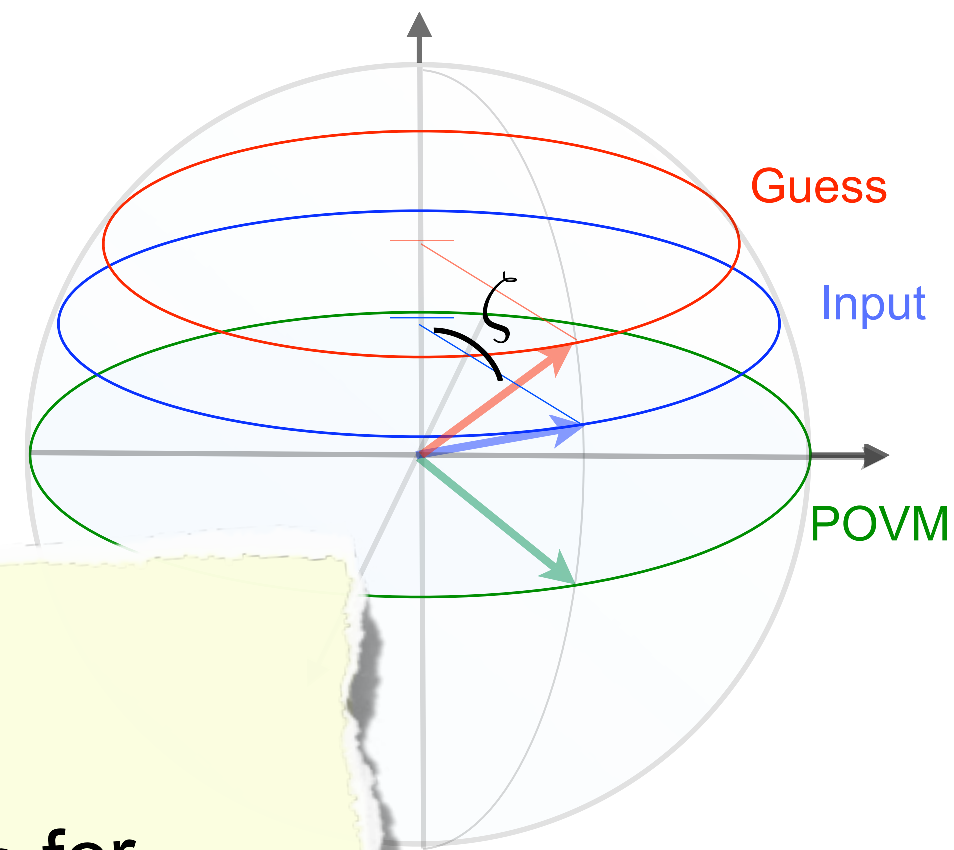
$$(1-a)(1-c) - |b|^2 = w^2, \quad ac - |b|^2 = u^2$$

Using Lagrange multipliers and after some algebraic manipulations we arrive to:

$$\begin{cases} a = \cos^2\zeta \\ c = \sin^2\zeta \\ b = \sqrt{ac} = 1/2\sin 2\zeta \end{cases} \qquad \zeta = \arctan \frac{r^2\sin^2\theta + \sqrt{(1-r^2)(4-r^2\sin^2\theta)}}{2r\cos\theta}$$

- Optimal strategy given by single-seed covariant POVM

$$\overbrace{2\,|+\rangle\langle+|}^{\text{POVM-element}} \otimes \overbrace{U_{y,\zeta}\,|0\rangle\langle 0|\,U_{y,\zeta}^{\dagger}}^{\text{guessed-state}} = \begin{pmatrix} \cos^2\frac{\zeta}{2} & \frac{1}{2}\sin\zeta & \cos^2\frac{\zeta}{2} & \frac{1}{2}\sin\zeta \\ \frac{1}{2}\sin\zeta & \sin^2\frac{\zeta}{2} & \frac{1}{2}\sin\zeta & \sin^2\frac{\zeta}{2} \\ \cos^2\frac{\zeta}{2} & \frac{1}{2}\sin\zeta & \cos^2\frac{\zeta}{2} & \frac{1}{2}\sin\zeta \\ \frac{1}{2}\sin\zeta & \sin^2\frac{\zeta}{2} & \frac{1}{2}\sin\zeta & \sin^2\frac{\zeta}{2} \end{pmatrix}$$




$$K = \begin{pmatrix} \cos^2\frac{\zeta}{2} & 0 & 0 & 0 \\ 0 & \sin^2\frac{\zeta}{2} & \frac{1}{2}\sin\zeta & 0 \\ \ldots \end{pmatrix}$$

(*)

- Numerical results indicate that single-seed covariant POVM is not optimal for d≥3, even for pure state (difference in 3rd digit).
- No known bounds on the minimal number of POVM elements (or seeds)

$$\mathcal{F} = \frac{1}{2}\Big(1 \ldots \, {}^{2}\theta)$$

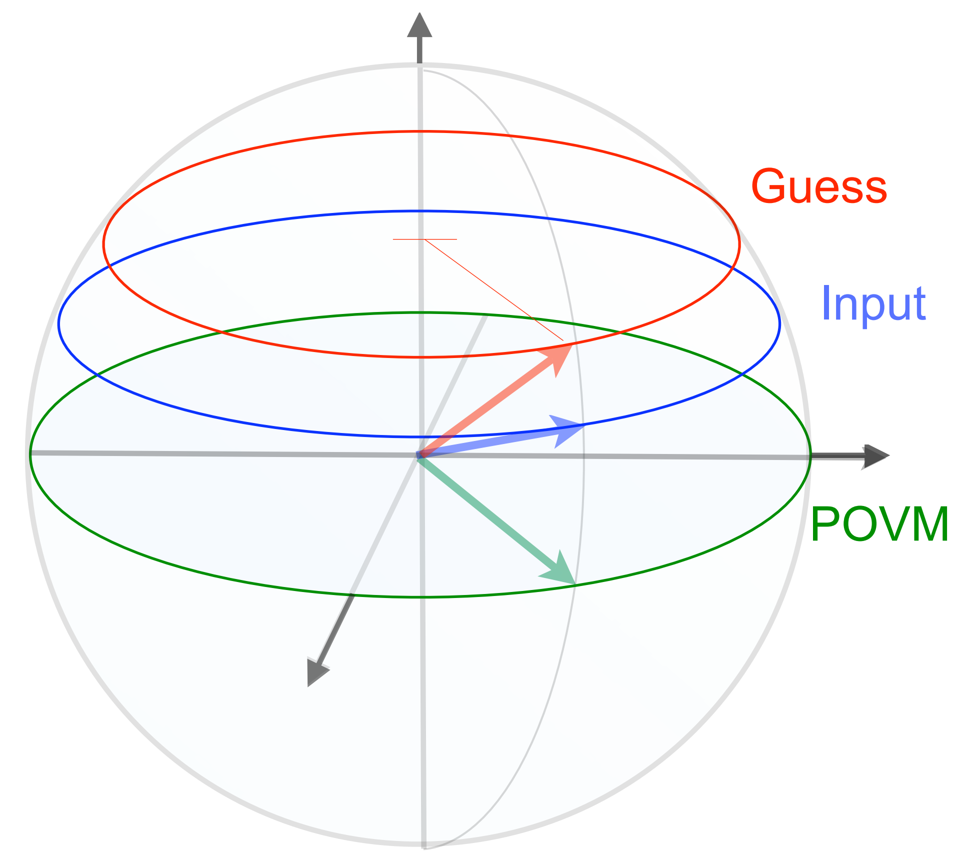
- Single-seed POVM (canonical phase-measurement) is optimal (*!)
- Guess does not belong to input family:
  - Guess is always pure.
  - Guess points in a different direction than input state.

$$\zeta \leq \theta$$

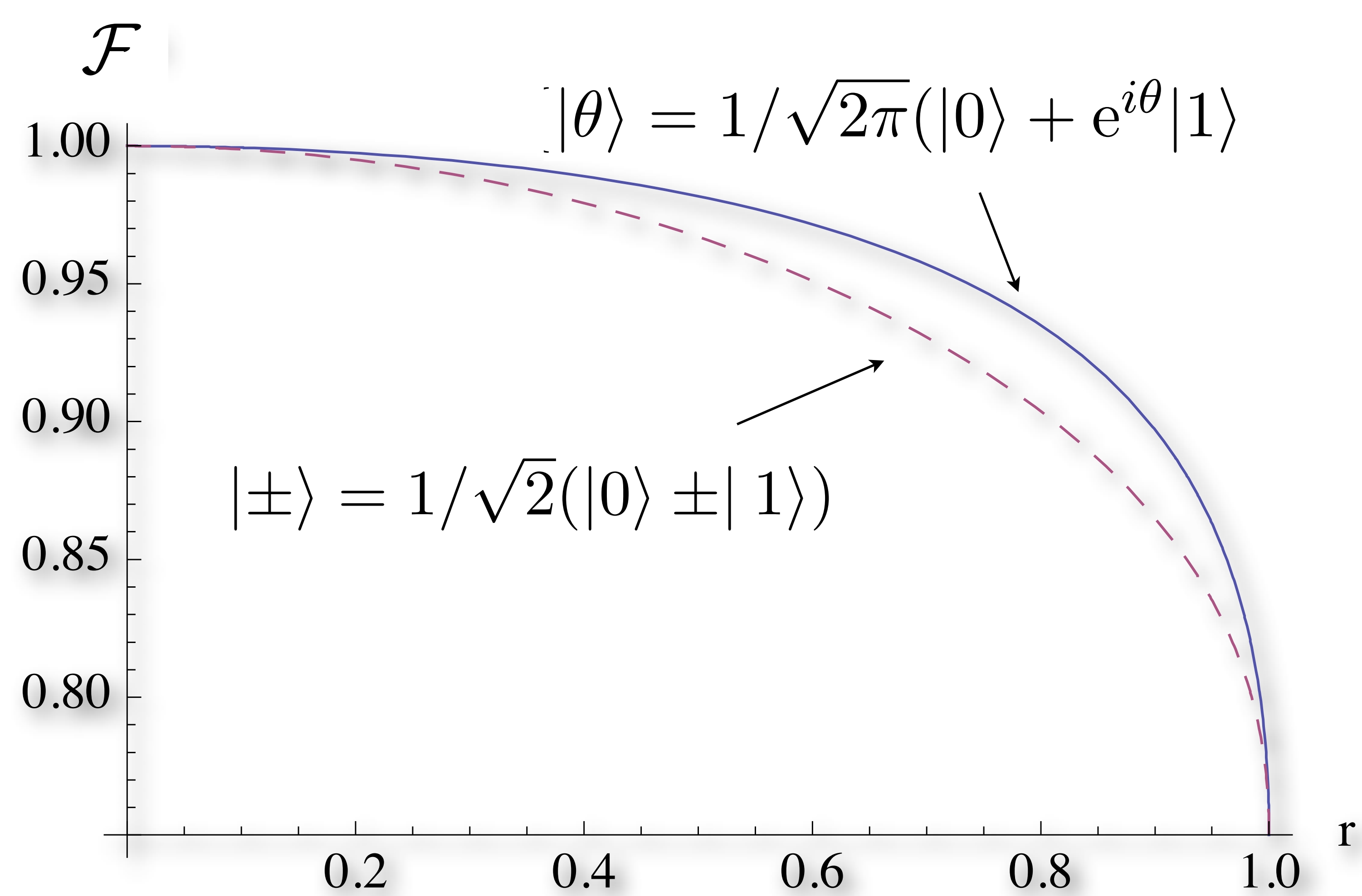
# Qubits

$$\mathcal{F} = \frac{1}{2}\left(1 + r\frac{\cos\theta}{\cos\zeta}\right)$$

$$\zeta = \arctan \frac{r^2 \sin^2\theta + \sqrt{(1-r^2)(4 - r^2\sin^2\theta)}}{2r\cos\theta}$$




- Pure states $\quad \mathcal{F} = \frac{1}{8}\left[7 + \cos(2\theta)\right]$
- Equatorial plane $\quad \mathcal{F} = \frac{1}{4}\left(2 + r^2 + \sqrt{4 - 5r^2 + r^4}\right)$
- Continuos POVM overcomes
  2-outcome S-G measurement.
- Mixedness improves classical Fidelity



# Semidefinite programming (SDP)

Minimize a linear *objective function* subject to *semidefiniteness* constraints involving symmetric matrices that are affine in the variables.

*Primal* problem:

$$p^* = \min_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \;\; \text{subject to} \;\; F(\mathbf{x}) = F_0 + \textstyle\sum_i x_i F_i \geq 0$$

*Dual* problem:

$$d^* = \max_Z -\mathrm{tr}(Z F_0) \;\; \text{subject to} \;\; Z \geq 0 \;\; \text{and} \; c_i = \mathrm{tr}(Z F_i)$$

- $d^* \leq p^*$ (equality if feasible point exist such that $F(\vec{\mathbf{x}}) > 0$ ).

- Pure states: $\mathcal{F} = \max_K \operatorname{tr}(K\rho_0 \otimes \rho_0)$ $\rho_0 = |\psi_0\rangle\langle\psi_0|$

$$\begin{cases} K \geq 0 \\ \operatorname{tr}_B K = \mathbb{1}_A \\ K \text{ invariant under bilateral } U \otimes U \\ K \text{ separable}^{*} \end{cases}$$

* Hierarchy of constrains based on PPT symmetric extensions.
(Doherty et al. PRA 2004)

Here we stay at first level of hierarchy, i.e., PPT $K^\Gamma \geq 0$. Hence, $\mathcal{F}^\Gamma \geq \mathcal{F}$

- Mixed states:

$$\mathcal{F} = \left(\mathrm{tr}|\sqrt{\rho_0}\sqrt{\rho_{\mathrm{av}}}|\right)^2$$ with $$\rho_{\mathrm{av}} = \int d\theta \sum_\chi p(\chi,\theta|\rho_0)\rho_{\chi,\theta}$$

The objective function becomes non-linear, but we can linearize it by making use of *Uhlmann's Theorem*:

$$\mathcal{F} = \max_{\Psi_{\mathrm{av}}} |\langle \Psi_0 | \Psi_{\mathrm{av}} \rangle|^2 = - \min_{\sigma_{\mathrm{av}}, K} \left( - \langle \Psi_0 | \sigma_{\mathrm{av}} | \Psi_0 \rangle \right)$$

where $|\Psi_0\rangle$ and $|\Psi_{\mathrm{av}}\rangle$ are *purifications** of $\rho_0$ and $\rho_{\mathrm{av}}$ respectively.

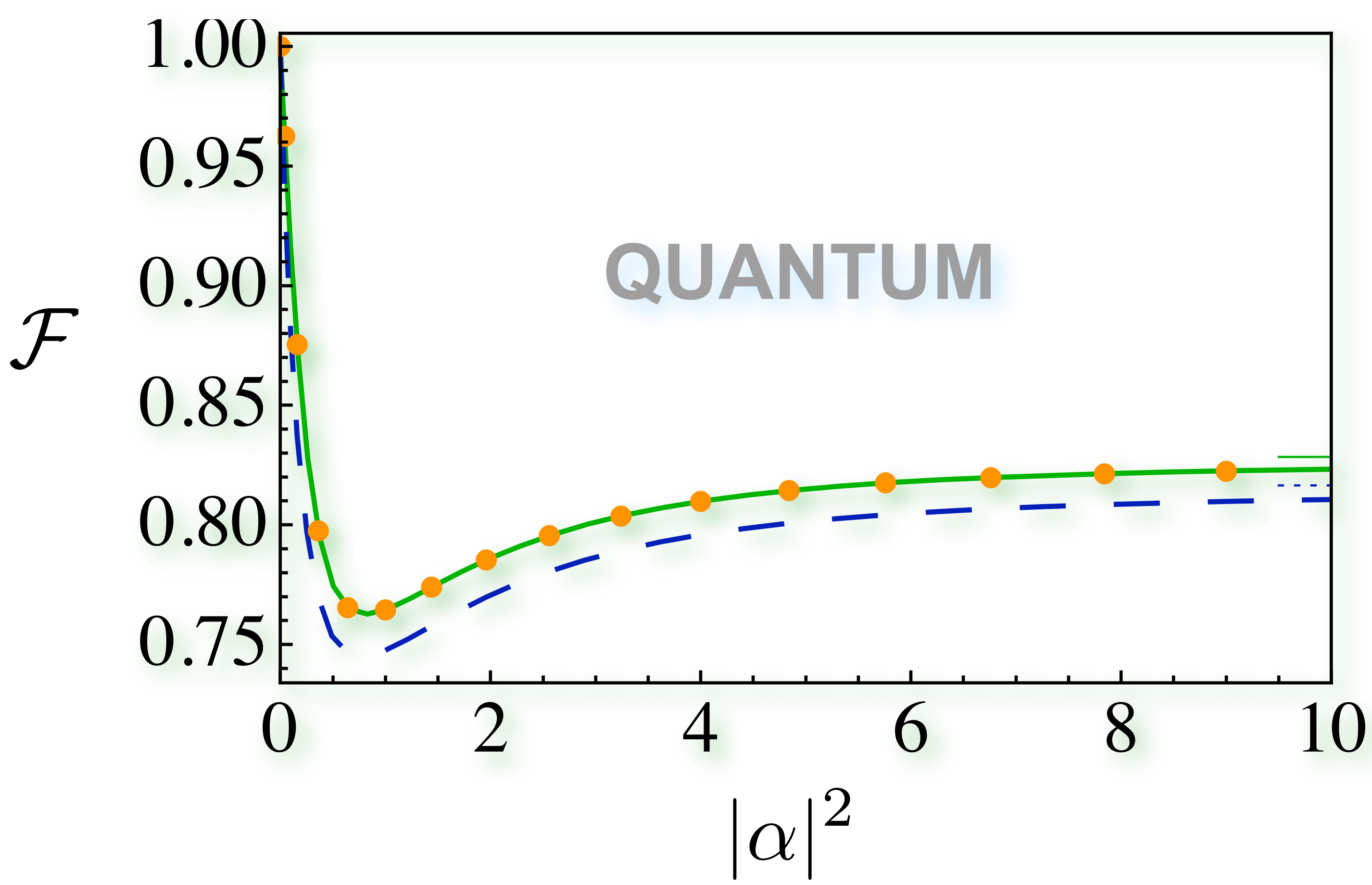
*$\rho_A = \mathrm{tr}_B \, |\Psi\rangle_{AB}\langle\Psi|$

$$\begin{cases} \text{(i) } \mathrm{tr}_B \sigma_{\mathrm{av}} = \rho_{\mathrm{av}} = \mathrm{tr}_A(\rho_0 \otimes \mathbb{1}\, K) \\ \text{(ii) } \sigma_{\mathrm{av}} \geq 0 \text{ and } \mathrm{tr}\,\sigma_{\mathrm{av}} = 1 \\ \text{(iii) the same conditions on } K \text{ as above} : K \geq 0, K \text{ separable}, \mathrm{tr}_B K = \mathbb{1}_A. \end{cases}$$

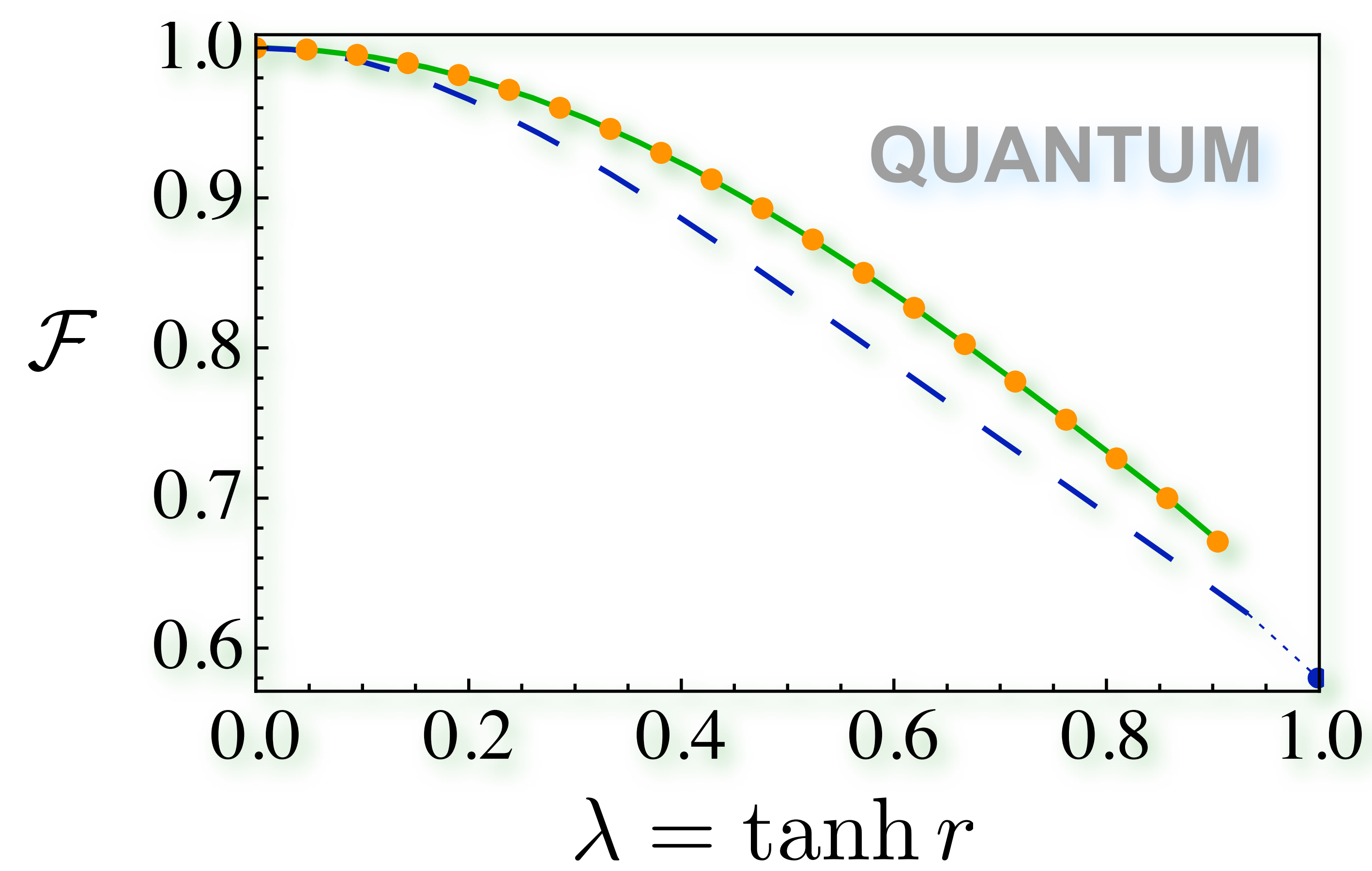
(purity condition $\sigma_{\mathrm{av}}^2 = \sigma_{\mathrm{av}}$ can be lifted)

# Results for pure CV gaussian states

Coherent States




Squeezed States




- SDP results (PPT constrain) and truncation: $|\alpha\rangle \approx \mathrm{e}^{-\alpha^2/2} \sum_{n=0}^{N} \alpha^n/\sqrt{n!}\,|n\rangle$
- Phase-measurement+optimal guess (max. eigenvalue of $A$ )
- Guess from input ensemble.

$$\mathcal{F} = \int \frac{d\phi}{2\pi} |\langle\xi|\alpha \mathrm{e}^{i\phi}\rangle|^2 |\langle\alpha|\alpha \mathrm{e}^{i\phi}\rangle|^2.$$

# Analytic results for asymptotic limits

$\alpha \gg 1$

- Restricted guess: $\mathcal{F} = \int \frac{d\phi}{2\pi} |\langle \xi | \alpha e^{i\phi}\rangle|^2 |\langle \alpha' | \alpha e^{i\phi}\rangle|^2$

$$\begin{cases} |\alpha'\rangle \text{ : coherent state} \\ |\xi\rangle = \sum_n |n\rangle \end{cases}$$

$$|\langle \xi | \alpha e^{i\phi}\rangle|^2 = |e^{-\alpha^2/2} \sum_n e^{in\phi} \frac{\alpha^n}{\sqrt{n!}}|^2 \simeq \sqrt{2\alpha^2/\pi} e^{-2\alpha^2\phi^2}$$

$$e^{-\alpha^2/2} \frac{\alpha^n}{\sqrt{n!}} = \sqrt{P_{\text{poiss}}(n)} \simeq \sqrt{\frac{1}{2\pi\alpha^2}} e^{-\frac{(\alpha^2-n)^2}{2\alpha^2}}$$

$|\langle \alpha | \alpha' \rangle|^2 = e^{-|\alpha-\alpha'|^2}$ where $|\alpha - \alpha'|^2 = \alpha^2[(\eta-1)^2 + 4\eta \sin^2(\phi/2)]$ with $\eta = \alpha'/\alpha$

$$\mathcal{F} = \int \frac{d\phi}{2\pi} |\langle \xi | \alpha \rangle|^2 \, |\langle \alpha | \alpha' \rangle|^2 = \sqrt{\frac{2\alpha^2}{\pi}} e^{-\alpha^2(\eta-1)^2} \int d\phi \; e^{-4\eta\alpha^2 \sin^2\phi/2} \, e^{-\alpha^2\phi^2} =$$

$$\simeq \sqrt{\frac{2\alpha^2}{\pi}} e^{-\alpha^2(\eta-1)^2} \int d\phi \, e^{-2\eta\alpha^2\phi^2} e^{-\alpha^2\phi^2} = e^{-\alpha^2(\eta-1)^2} \sqrt{\frac{2}{2+\eta}} \xrightarrow{\alpha\to\infty} \boxed{\sqrt{\frac{2}{3}}}$$

Optimal Guess: $\eta_{\text{opt}} \xrightarrow{\alpha\to\infty} 1 \Rightarrow \boxed{\alpha' = \alpha}$

# Analytic results for asymptotic limits

- Optimal guess for phase-measurement: $\mathcal{F} = ||A||_\infty = \lim_{p\to\infty} (\mathrm{tr} A^p)^{1/p}$

$$A = \int \frac{d\phi}{2\pi}\, p(\chi|\phi)\, |\alpha e^{i\phi}\rangle\langle\alpha e^{i\phi}| = \int \frac{d\phi}{2\pi}\, |\langle\xi|\alpha e^{i\phi}\rangle|^2\, |\alpha e^{i\phi}\rangle\langle\alpha e^{i\phi}|$$

$$(||A||_p)^p = \mathrm{tr} A^p = \int \prod_{j=1}^{p} d\phi_j\, p(\chi|\phi_j) \langle\alpha_j|\alpha_{j+1}\rangle, \qquad \alpha_{p+1} \equiv \alpha_1$$

$$\langle\alpha_i|\alpha_j\rangle \approx \exp\{-\alpha^2[i(\phi_i - \phi_j) + 1/2(\phi_i - \phi_j)^2]\}$$

$$\mathrm{tr} A^p \simeq \left(\frac{2\alpha^2}{\pi}\right)^{p/2} \int d^p\phi\, e^{-\frac{\alpha^2}{2}\phi^t \cdot C_p \cdot \phi} = \frac{2^p}{\sqrt{\det C_p}}, \qquad C_p = \begin{pmatrix} 6 & -1 & & & -1 \\ -1 & 6 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 6 & -1 \\ -1 & & & -1 & 6 \end{pmatrix}$$

Expanding in minors along the first row:

$$\det C_p = 6 \det B_{p-1} - 2 \det B_{p-2} - 2$$

$$B_p = \begin{pmatrix} 6 & -1 & & & 0 \\ -1 & 6 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 6 & -1 \\ 0 & & & -1 & 6 \end{pmatrix}$$

The following recursive relation holds:

$$\det B_p = 6 \det B_{p-1} - \det B_{p-2}$$

with $B_0 = 1, \; B_1 = 6$

→ $B_p = U_p(6/2)$

Chebyshev polynomials:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$
with $T_o(x) = 1, T_1(x) = x$ — 1rst kind

$$U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x)$$
with $U_o(x) = 1, U_1(x) = 2x$ — 2nd kind

$$\begin{aligned} \det C_p &= \det B_p - \det B_{p-2} - 2 = \\ &= U_p(6/2) - U_{p-2}(6/2) - 2 = 2[T_p(6/2) - 1] \end{aligned}$$

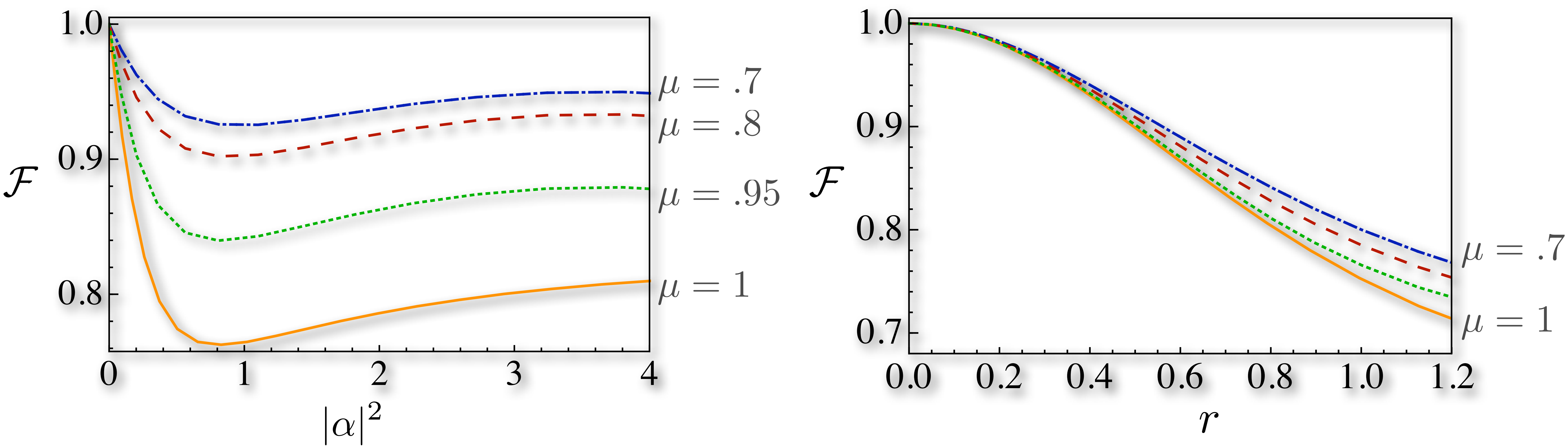
$$T_n(x) = \frac{1}{2}[U_n(x) - U_{n-2}(x)] \qquad T_n(x) = \frac{1}{2}[(x + \sqrt{x^2-1})^n + (x - \sqrt{x^2-1})^n]$$

$$\mathcal{F} = ||A||_\infty = \lim_{p\to\infty} 2(\det C_p)^{\frac{1}{2p}} = 2 \lim_{p\to\infty} [(3 + \sqrt{3^2-1})^p + (3 - \sqrt{3^2-1})^p]^{-\frac{1}{2p}}$$

$$= 2(3+\sqrt{8})^{-1/2} = 2(\sqrt{2}-1) \approx 0.8284$$

Difference between restricted/unrestricted guess persist in assymptotic regime!

# • Mixed gaussian states




- Benchmark becomes higher with mixedness
  (for same displacement and squeezing parameters)

- For phase-measurement & guess in $\Omega$, the effect is the opposite
  ($\mathcal{F}$ decreases with $\mu$).

# Conclusions

- Benchmarks for CV quantum storage & teleportation experiments
  - Phase covariant family of test states. Easy to implement.
  - Valid for mixed test states.
  - quantum state estimation revised.

Thank you for your attention