ᘔ' Ɛ' 8' ϛ' ¡' ᘔ '' ᘔ' Ɛ' 'ɯ ∂ᵷ 8' ϛ' '' 'ɐ '' '' '' 'ɟ '

,j ,, ,l ,, ,, ,, ,s ,, ,m ,, ,, ,, ,h ,k ,s @5 ,, ,b ,, ,k ,, ,2 ,a ,, ,j ,, ,, ,, ,,

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,, ,, ,- ,, ,, ,, ,, 0l ,, ,, ,, ,, ,3 ,, ,, 3, ,, ,a ,i ,l ,, (62)(63). ΓΙΓΙΓ7, ΓbΓGΓkΓ,HsΓ5B'ΓGΓΙΓkΓl9gΓK, Γ:Γ‡ΓNΓhΓ&ΓJB EΙΓJ8=Γ]Γ,5ΙΓ-ΓkΓHΓ8BΓiΓ:Γ5B'ΓJΓbΓGΓkΓ6aΓΙΓH-ΓdΓbΓGΓkΓ,BΓNΓGΓΙΓkΓH-ΓONΓΓΓ0l CW9Γk

Γ3ΓNA0cΓ0J2ΙΓK=ΓΙΓkΓhΓ&ΓKEΙΓdDΙΓ6h4VΓdDΙΓNA0cΓHΓΙΓ&4ΓEΙΓ‡FiBΙΓ(ΓkΓ3ΓHΓ,ΓGΓ-Γk. Γ˜Γ:, ϵ αrαα Dαα ααααα NBgJP.αα N>α Aααh ααd Dαα BR ˜ααFααk ααααααk αααααααα, $\epsilon \to +0$ Γ6K8BΓr9MΓ(ΓΙΓH-ΓNBgJP.9 ΓN0U LΙΓGGE,ΓJΓdDΓNΙΓr9MΓ(ΓkΓ3ΓHΓOEΙΓdDΙΓ6B˜Γ7ΓHΓΙΓkΓ9MΓ(ΓkΓ3ΓHΓ,=ΓMΓk. Γ7ΓΙΓ,ΓCΓHΓe5-ΓNΓ2ΓDΓNNΙΓNT0l CW‡FiEΙΓdDΙΓ,6h4VΓdDΙΓN6K8BΓH7ΓEBΙΓ(ΓkΓ3ΓHΓ,ΓGΓ-ΓJΓΙΓ9MΓ(ΓiΓlΓk. Γ˜Γ5UΓK5B'ΓJΓbΓGΓkΓNΓH-ΓNΓhΓ&ΓK NΙΓΓ,0l CW9ΓkΓl9gΓKDΓΙΓOEΙΓdDΙΓr6h4VΓdDΙΓN6K8BΓH7ΓEBΙΓ(ΓkΓ3ΓHΓ,ΓGΓ-Γk. 6K8BJ, I[ΓKΓΙΓ1ΓkF1MΓM9MΓΙΓK ΓΙΓΙΓHΓb§4.2 ΓG8‡ΓkΓhΓ&ΓKF1ΓoΓNΓ2ΓDΓN6K8BΓ0Γr9MΓ(ΓkΓ3ΓHΓ,ΓGΓ-Γk. Γ‡Γ7ΓHsΓ5B'ΓNΓl9gΓKΓ0l HΙΓKΓONΙΓΓ, 0[ΓkΓ3ΓHΓ,F1MΓM3NΓ‡FaΓiΓl (87)(88), Γ5B'ΓJΓbΓGΓkΓb7ΓΙΓO5B'ΓJΓbΓGΓkΓ6aΓΙΓl9gΓO‡ΓNΓhΓ&ΓJNΙΓΓ,0l CW9 ΓkΓ3ΓHΓ,3NΓ‡FaΓiΓlΓk(89)-(92). Γ7Γ‡F7ΓJΓ,Γi, KJ8ΓrFΙΓaΓJ, ΓkΓhΓ&ΓKBgJP.9 ΓNΓl9gΓNΓhΓ&ΓKNΙΓΓN0cΓΙΓr6h4V ΓdDΙΓHEΙΓdDΙΓNA0cΓK7kΓSIUΓ1ΓkΓHΓΙΓ&rΓaΓ,ΓGΓ-ΓJΓΓ.

Γ‡ΓNΓ[Γ‡BgJP.9 7ΙΓNΓ2AΓK8BΓCEMΓM9ΓJ5DOΙΓr §3 ΓGΓ07ΓCΓΓ. §3.3 ΓGΓO1&B&m9 ΓH8 B&m9 ΓrF1EyΓKΓ2AΓ9Γk OHAHΓΙΓG2 ΓΙΓNA2aMDΙΓb07ΓCΓΓ. Γ5B'ΓJΓbΓGΓkΓGΓNBgJP.9 7ΙΓ2AΓN2ΙΓGΓN9bΓΙΓNA2aOΙΓOFu[4] ΓKΓhΓI5DOΙΓ5Γl ΓHΓΙΓkΓ,K9FΓG07Γ&ΓhΓ&ΓJHsΓ5B'ΓJΓbΓGΓkΓGΓNBgJP.9 7ΙΓ2AΓN2ΙΓGΓN9bΓΙΓNA2aOΙΓOCxΓΓΓNCNΓk8BΓΙΓJΓ5ΓlΓHΓΓ ΓJΓΓ. Γ7Γ‡F7, ΓDGΓJΓ,ΓiΓ3ΓlΓKDΓΙΓHΓOCΙΓ.2DG‡J8B3&ΓrΓ[ΓHsΓI5aΓaΓkΓ3ΓHΓ,ΓGΓ-Γ::Γ8eΓN2] BΙΓHΓJΓk.

Γ‡Γ7ΓF§3.4 ΓGΓOBgJP.9 7ΙΓNΓ2AΓKΓΙΓ1ΓkD6M8ΙΓ-ΓKDΓΙΓHΓb(Γ1ΓHΓΙΓΙΓΓ. ΓbΓGΓkΓ,Γ5B'ΓGΓΙΓkΓH-ΓKΓOBgJP :9 7ΙΓNΓ2AΓKDΓΙΓHΓOAIBΓ(ΓsΓHΓnΓΓΓ(Kullback-Leibler ΓNdivergence) ΓNC1D4Γ-Γ‡Fi8B3&ΓrF3ΓΙΓ3ΓHΓ,ΓGΓ-Γk [5, 6] ΓΙΓa, MSE ΓH0[ΓJΓIDΓ6M8ΙΓ-ΓG5ΙΓ-ΓJΓΓ. Γ7Γ‡F7ΓJΓ,Γi, ΓbΓGΓkΓ,HsΓ5B'ΓGΓΙΓkΓl9gΓOBgJP.9 7ΙΓNΓ2AΓKΓ ΓΙΓHΓbD6M8ΙΓJΓdDΓNΙΓ,B8ΓΓ9ΓkΓ3ΓHΓ, Ibragimov and Has'minskii [3] ΓKΓhΓΙΓIE&F5ΓlΓHΓΙΓk. Γ3Γ3ΓGΓOK9FΓG 9ΓΓCΓDΙΓ02=ΓHD6M8ΙΓ-ΓHN4Γ78ΓKDΓΙΓF=ΓRΙΓk.

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,, ,@,, , p^n $\beta\beta\beta\beta\beta\beta J\beta\beta\beta$ p αNn $\alpha ID\alpha NF\alpha N\alpha F\alpha\alpha\alpha J\alpha I\alpha\alpha\alpha\alpha\alpha k$: $\Gamma\Pi\Gamma\mathrm{K}f$ $\alpha r\,\Omega_1$ $\Gamma{+}\Gamma$

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$$-\lim_{n\to\infty} \log\left(\beta(p_n, T_n) + \exp(-L_n r)\beta(q_n, T_n)\right) \geq \sup_{0<s<1}$$

$\lim_{n\to\infty} \frac{1}{L_n q_n(C_n)} \int_{C_n} \log \frac{\partial p_n}{\partial q_n}(\omega) q_n(d\omega)$ LN8LLr2LDLL9Lk L3LlLiLNLr7oLN2L

$\lim_{n\to\infty} \frac{1}{L_n} I^s(p_n\|q_n)$ L, s ααα∂αα∂ααEy∕αα∂α\αααα: L=LNBLL2LDLN6k8B $\lim_{n\to\infty}$

☐ βCβt βs βββββk. L3LNLH-,0

,3 ,3 ,, $\psi_r(s) := sr - \log \int_C \exp\left(s \log \frac{\partial q}{\partial p}(\omega)\right) p(d\omega)$ ΓH9ΓkΓH$\phi_r(t)$ ΓO1Γ4ΓΓt (ΓeΓKΓI) ΓHΓJΓk Γ5ΓiΓK(26) ΓNΓr 7oΓhΓΓ$\psi'_r(1) \leq 0$ ΓHΓJΓkΓ7ΓΓ,ΓCΓF

$$-\lim_{n\to\infty} \frac{1}{n} \log \beta(p, T(r)_n) = \sup_{0\leq s\leq 1} \phi_r(s) = \sup_{0\leq s\leq 1} rs + I^s(q\|p).$$

ααα𝒥αα(26) ΓrFΓΓk ☐

☐ 3 ΓO0J2ΓΓNΓhΓ&ΓK0l HΓ2=ΓGΓ-Γk

JdBΓ4 p_n, q_n βr βββββuββ Ω_n ββββββββJβββββββ, $C_n := \mathrm{supp}(p_n) \cap \mathrm{supp}(q_n)$ ΓeΓGp_n, q_n ββββββββββ∂βBβββBβ βββββk. Γ5ΓiΓKL_n βr LββBBgββββ,β βββkCβββA

$$-\inf_{R\in\Lambda'} I(R) \le \liminf_{n\to\infty} \frac{1}{L_n} \log p_n\{X_n \in \Lambda'\}.$$

,3 ,, ,, ,- $\Gamma \subset \mathbb{R}$ 0J2ΓNTEyΓ0Γ,Γ.N)Γ9Γk

$$\limsup_{n\to\infty} \frac{1}{L_n} \log p_n\{X_n \in \Gamma\} \le -\inf_{R\in\overline{\Gamma}} I(R). \tag{32}$$

$\overline{\Gamma}$ ΓOΓ ΓNDJqΓrI=Γ9Γ3ΓHΓH9Γk. Γ3Γ3ΓG$R \in \mathbb{R}$ $I(R)$ Γ0J2ΓΓGD5AΓ7ΓΓ. Γ3ΓNΓH-, ϕ (a, b) ΓNM8BΓ-Γ+Fi $I(R)$ ΓOOB3ΓHΓJΓk.

$$I(R) := \sup_{s\in(a,b)} (sR - \phi(s)). \tag{33}$$

Γ5ΓiΓK$\phi(s)$ Γ,3+6h4V(a, b) ΓeΓGHyJ, 2DG+GFΓΓkΓH-, Γ3ΓNΓH-=8g $\Lambda := \{\phi'(s) | s \in (a, b)\}$, Λ ΓN3+ It J, =8g Λ'

support Γ,=EJΓiΓJΓΓΓHΓΓ&0ULΓΓGFNHsΓ5B'ΓJΓb

ΓΓΓdDΓrBgJP.9 7ΓΓΓ2AΓNΓkEΓΓ+Fi07Γ&

3.1

ΓdDCM,ΓΓCMhΓIBgΓ-ΓΓΓJΓk8m9 ΓJ1&B8m9 ΓKΓH

ΓDΓk8m9 ΓrJLΓ1ΓF5DOΓΓ7, ΓbΓGFkΓ,HsΓ5B'ΓNΓ19gΓKΓ0l HΓΓKΓO

Γ8ΓaΓK,ΓdDΓNL$T, \epsilon > 0$ ΓKBPΓ7ΓF$\beta^+(T, \theta, a), \beta^-(T, \theta, a)$ Γr0J2

$$\beta^+(T, \theta, a) := -\liminf_{n\to\infty} \frac{1}{n} \log p_\theta^n (T_n \ge a), \quad \beta^-(T$$

$\beta(T, \theta, \epsilon)$ Γr $\beta(T, \theta, \epsilon) := \min\{\beta^+(T, \theta, \theta + \epsilon), \beta^-(T, \theta, \theta -$

$\beta^-(T, \theta, a)$ ΓO

0J2Γ$g(x)$ Γr $x \to 0$ ΓG0 ΓKΓIB+F9ΓkOB34ΓΓtΓHΓ70J2ΓΓNNΓr

$$\alpha^+(g, T, \theta) := \lim_{\epsilon\to+0} \frac{\beta^+(T, \theta, \theta + \epsilon)}{g(\epsilon)}, \quad \alpha^-(g, T, \theta) := \lim_{\epsilon\to+0}$$

Γe5- 2 ΓDΓNNΓ,B8ΓΓ7θ_0 Γ

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,j , **7** **2((** $g(x) = x^2$ Γ

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2(($\gamma_1 = \gamma_2 = 2$ FH-FOC(g, θ) FOR

$g(x) = -x^2 \log|x| ---||$

fig. 3

,j ,, ,l ,, ,, ,, ,s ,, ,m,, ,, ,, ,1 ,, @-,h ,j 0, 2, ,, ,0 ,, ,3 ,1 ,k .

$$
\begin{aligned}
& -nI^s(p_\theta \| p_{\theta'}) \\
& \le \log\Bigg(\left(p_\theta^n\left(T_n \le \theta\right)\right)^s \left(p_{\theta'}^n\left(T_n \le \theta\right)\right)^{1-s} + \left(p_\theta^n\left(\theta' < T_n\right)\right)^s \left(p_{\theta'}^n\left(\theta' < T_n\right)\right)^{1-s} \\
& \qquad\qquad + \sum_{i=1}^m \left(p_\theta^n\left(\theta_{i-1} < T_n \le \theta_i\right)\right)^{1-s} \left(p_{\theta'}^n\left(\theta_{i-1} < T_n \le \theta_i\right)\right)^{1-s} \Bigg) \\
& \le \log\left(\left(p_{\theta'}^n\left(T_n \le \theta\right)\right)^{1-s} + \left(p_\theta^n\left(\theta' < T_n\right)\right)^{1-s} + \sum_{i=1}^m \left(p_\theta^n\left(\theta_{i-1} < T_n\right)\right)^s \left(p_{\theta'}^n\left(T_n \le \theta_i\right)\right)^{1-s} \right) \\
& \le \log\Bigg(+ \exp\left(-ns\left(\beta^+(T,\theta,\theta') - \epsilon\right)\right) + \exp\left(-n(1-s)\left(\beta^-(T,\theta',\theta) - \epsilon\right)\right) \\
& \qquad\qquad + \sum_{i=1}^m \exp\left(-n\left(s\beta^+(T,\theta,\theta_{i-1}) + (1-s)\beta^-(T,\theta',\theta_i) - \epsilon\right)\right) \Bigg) \\
& \le \log\left((m+2)\exp\left(-n\left(\min_{1\le i\le m} s\beta^+(T,\theta,\theta_{i-1}) + (1-s)\beta^-(T,\theta',\theta_i) - \epsilon\right)\right)\right) \\
& = \left(\log(m+2) - n\left(\min_{1\le i\le m} s\beta^+(T,\theta,\theta_{i-1}) + (1-s)\beta^-(T,\theta',\theta_i) - \epsilon\right)\right).
\end{aligned}
$$

N≯J$\alpha\alpha r$ n $\alpha\alpha\alpha d\alpha C\alpha\alpha\alpha e\alpha\alpha n \to \infty$ −−−−−‖+−,

$$
I^s(p_\theta \| p_{\theta'}) \ge \left(\min_{1\le i\le m} s\beta^+(T,\theta,\theta_{i-1}) + (1-s)\beta^-(T,\theta',\theta_i) - \epsilon\right)
$$

(((((($\epsilon > 0$ ΓOGI0UΓGFIΓkΓ+Fi,

$$
I^s(p_\theta \| p_{\theta'}) \ge \min_{1\le i\le m} s\beta^+(T,\theta,\theta_{i-1}) + (1-s)\beta^-(T,\theta',\theta_i)
$$

ΓrFIΓk$m \to \infty$ −−−−−‖+−(37) ΓrFIΓk. □

2 ΓrΓ(Γ9ΓIΓaΓK$t(T) := \lim_{\epsilon\to+0} \dfrac{\beta^-(T,\theta,\theta+\epsilon)}{\beta^+(T,\theta,\theta-\epsilon)}$ ΓHDI5AΓ7, 0J2ΓΓNJdBIΓr=IHvF9Γk.

JdBΓ**7** $t(T) = t$ $\alpha\alpha\alpha\alpha$, T $\alpha r\theta$ $\alpha\alpha\alpha Ng$-LD 7Γ6I=I6&JIΓdDINIΓHΓ9Γk. Γ3ΓNΓH-0J2ΓΓNTEyΓ0ΓrFIΓk.

$$
\begin{aligned}
\alpha^+(g,T,\theta) &\le \inf_{\{s|0<s<1\}} \frac{\displaystyle\lim_{\epsilon\to 0} \frac{I^s(p_\theta \| p_{\theta+\epsilon})}{g(\epsilon)}}{\displaystyle\lim_{\epsilon\to 0} \frac{\inf_{\{\eta|0\le\eta\le 1\}} sg(\eta\epsilon) + (1-s)tg((1-\eta)\epsilon)}{g(\epsilon)}} \\
&= \inf_{\{s|0<s<1\}} \lim_{\epsilon\to 0} \sup_{\{\eta|0\le\eta\le 1\}} \frac{I^s(p_\theta \| p_{\theta+\epsilon})}{sg(\eta\epsilon) + (1-s)tg((1-\eta)\epsilon)}.
\end{aligned} \tag{38}
$$

$\alpha\alpha\alpha i\alpha\alpha g$ $\alpha\alpha\alpha z\alpha X\alpha t\alpha\alpha\alpha\alpha k\alpha\alpha\alpha$(38) ΓO0J2ΓΓNΓhΓ&FKJΓ7AΓGF-Γk.

$$
\alpha^+(g,T,\theta) \le \inf_{\{s|0<s<1\}} \frac{1}{\min\{s,(1-s)t\}} \lim_{\epsilon\to 0} \frac{I^s(p_\theta \| p_{\theta+\epsilon})}{g(\epsilon)}. \tag{39}
$$

ΓZLΓ Γ3Γ3ΓGJdBΓ6 ΓrMΓIΓkΓH, 0J2ΓΓNΓ0Γ,FIΓiΓlΓk.

$$
\frac{1}{g(\epsilon)} \inf_{\{\eta|0\le\eta\le 1\}} s\overline{\beta}^+(T,\theta,\theta+\eta\epsilon) + (1-s)t\overline{\beta}^-(T,\theta+\epsilon,\theta+\eta\epsilon) \le \frac{I^s(p_\theta \| p_{\theta+\epsilon})}{g(\epsilon)}. \tag{40}
$$

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,5 ,i ,, $1 < \gamma_1 = \gamma_2 < 2$ ΓΝΓl9g0J2ΓΓΝΓ0Γ,FΓΓiΓlΓk

$$\begin{aligned}
&\alpha^+(g,T,\theta)\\
&\le \inf_{\{s|0<s<1\}} \frac{A_1 s(1-s(\gamma_1-1))B(s+\gamma_1(1-s),2-\gamma_1)+A_2(1-s)(1-(1-s)(\gamma_1-1))B(1-s+\gamma_1 s,2-\gamma_1)}{\gamma_1 \inf_{\{\eta|0\le\eta\le1\}} s\eta^{\gamma_1}+(1-s)t(1-\eta)^{\gamma_1}}.
\end{aligned} \tag{45}$$

Γ∓F7ΓF $\gamma_1 = \gamma_2 = 2$ ΓΝΓΗΓ-ΓΚΓDΓΓΓΗΓO $g(x) = -x^2 \log x$ αααααkαααJα<αΝ<ααFα∂αiααk:

$$\alpha^+(g,T,\theta) \le \begin{cases} \frac{A_1+A_2}{2} & t < 1 \\ \frac{A_1+A_2}{2t} & t \ge 1. \end{cases} \tag{46}$$

$\gamma_1 = \gamma_2 > 2$ ΓΝΓΗΓ-ΓOJdBΓ8 ΓrΓ2ΓΗΓΝΓ3ΓΗ FΓOΓK $A_1 = A_2, t = 1$ ΓΝΓΗΓ-ΓO0J2ΓΓΝΓ0Γ,Γ.Ν)Γ9Γk

$$\alpha^+(g,T,\theta) \le \begin{cases} \frac{2(1-\gamma_1)}{\gamma_1} A_1 B\left(\frac{1+\gamma_1}{2}, 1-\gamma_1\right) & 0 < \gamma_1 < 1 \\ 2A_1 & \gamma_1 = 1 \\ \frac{1}{\gamma_1} A_1 2^{\gamma_1}\left(1-\frac{\gamma_1-1}{2}\right) B\left(\frac{1+\gamma_1}{2}, 2-\gamma_1\right) & 1 < \gamma_1 < 2 \\ A_1 & \gamma_1 = 2. \end{cases} \tag{47}$$

ΓˆΤΓΓ $\gamma_1 < \gamma_2$ Γ ΓΓΓ ΓΓΓ

0, 2, (48) ΓrΓ(Γ9. $h(\epsilon, \eta) := \dfrac{-s(\eta\epsilon)^2 \log(\eta\epsilon) - (1-s)t((1-\eta)\epsilon)^2 \log((1-\eta)\epsilon)}{-\epsilon^2 \log \epsilon}$ ααααα Γ3Γ3ΓG$h(\epsilon, \eta)$ ΓNDΓ5AΓ0ΓN J, Jl ΓOΓ5ΓKΓJΓOΓHΓIΓkΓ3ΓHΓKΓm0ΓUΓIΓh. $h(\epsilon, \eta) \geq s\eta^2 + (1-s)(1-\eta)^2$ Γ

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$$p^n_\theta(T_n \le \theta - \epsilon) \le \left(\int_{a}^{b-\frac{\epsilon}{1-\lambda}} f(x)\,dx\right)^n. \tag{55}$$

(∫((. LPLCLF(25) LrFLL(23) LKDLLHPLMMKE3LL3LH LCL-LF L'LL(26) LPL0135LLML0L'E3LLF

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$$g(\underline{\omega}_n, \overline{\omega}_n) := \begin{cases} \int_{\underline{\omega}_n}^{\overline{\omega}_n} f(x)\,dx & \overline{\omega}_n \ge \underline{\omega}_n \\ 0 & \overline{\omega}_n < \underline{\omega}_n \end{cases}$$

ββββββββ, 0J2ΓΓNΓ0ΓrFΓΓk

$$p^n_\theta(T_n \le \theta - \epsilon) = \int_{\check{\theta}(\lambda) \le \epsilon} n(n-1) g(\underline{\omega}_n, \overline{\omega}_n)^n f(\underline{\omega}_n) f(\overline{\omega}_n)\, d\underline{\omega}_n\, d\overline{\omega}_n. \tag{56}$$

$f(\underline{\omega}_n) f(\overline{\omega}_n)$ 5ZΓS$g(\underline{\omega}_n, \overline{\omega}_n)$ ΓNOIB3Γ-ΓhΓΓ

$$\begin{aligned} \lim_{n\to\infty} \frac{1}{n} \log p^n_\theta(T_n \le \theta - \epsilon) &= \sup_{\check{\theta}(\lambda) \le \epsilon} -\log g(\underline{\omega}_n, \overline{\omega}_n) \\ &= -\log\left(\int_a^{b-\frac{\epsilon}{1-\lambda}} f(x)\,dx\right) \end{aligned}$$

(□ □

Hupeɪ[10] LKLPLIDRAL2LILIL ʻʻ KI∂ELCLOHaL2B,LʻILI∂ᵍLKLPL3LILᴵ3HDI

−−‖ ΓIEI

,, ,, ,, ,, ,j ,, ,, 8, ,, ,l ,k ,, ,j ,, $\tilde{\theta}_{c,\epsilon} = \{\tilde{\theta}_{n,c,\epsilon}\}$ −−−

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,r ,o **(A.5)** ($\gamma_1 = \gamma_2 = 2$ ΓHΓJΓΓ, LIEIHfΓdDINIΓ,B8ΓΓ9ΓkΓl9gΓKΓO=∓,ΓJ2rΓOΓ,=PMIFHΓIΓJΓIΓ,Γ

(((. (((8(

$$h^{+}\left(\hat{\theta}\left(\frac{1}{2}\right),\theta\right)(x) = 2f(p)x + 2\left(-f_1(p) + f_2(p)\right)x^2$$

(25),(23) [illegible] $f(a) = f(p)$ [illegible]
[illegible] $\hat{\theta}(\lambda)$ [illegible] $= 1/2$ [illegible]
3 [illegible]

,, @.,j ,, ,, . Γ7ΤΙΓ,ΓΟΓFGΙDUΓN$\epsilon > 0$ ΓKΓDΓΙΓF0J2ΙΓNΓ0ΓrFΙΓk.

$$\beta^+\left(\check\theta\left(\frac{1}{2}\right)',\theta,\theta+\epsilon\right)\begin{cases}=-\log(1-2\epsilon), & \text{if } 2\epsilon\le\theta\\ \ge-\log(1-2\epsilon), & \text{if } 0<\theta<2\epsilon\\ =\infty, & \text{if } \theta=0\\ \ge-\log(1-2\epsilon), & \text{if } -\epsilon<\theta<0\\ =-\log(1+\theta), & \text{if } -2\epsilon<\theta\le-\epsilon\\ =-\log(1-2\epsilon), & \text{if } \theta\le-2\epsilon.\end{cases}$$

$\alpha\alpha N$>$\alpha g\alpha\alpha$(35) ΓGl MMΙB∓F7ΤΗΓΙΓJΓΙΓNΓGDΙMI9 ΓOE,MΓGF-ΓJΓΙΓ, $g(x)=|x|$ −−−‖−∈−−−−−+−−−−−‖

$$\alpha^+\left(g,\check\theta\left(\frac{1}{2}\right)',\theta\right)=\begin{cases}\infty & \text{if } \theta=0\\ 2 & \text{if } \theta\ne0.\end{cases}\tag{68}$$

F1MMΙK7ΤF0J2ΙΓNΓ0ΓrFΙΓk

$$\alpha^-\left(g,\check\theta\left(\frac{1}{2}\right)',\theta\right)=\begin{cases}\infty & \text{if } \theta=0\\ 2 & \text{if } \theta\ne0.\end{cases},\quad \alpha\left(g,\check\theta\left(\frac{1}{2}\right)',\theta\right)=\begin{cases}\infty & \text{if } \theta=0\\ 2 & \text{if } \theta\ne0.\end{cases}\tag{69}$$

Γe5-ΓNNc1 ΓGFOLD 7Γ6I=Γ6&JΙΓ-ΓrKΤΙΓ5ΓJΓΙΓdDΙNL$\check\theta\left(\frac{1}{2}\right)'$ −−(54),(55),(68) 5ZΓS(69) ΓhΓΓ$\alpha(x,\check\theta\left(\frac{1}{2}\right)',\theta)$ ΓK ChLΙΓ9Γk8BΓΓ, $\check\theta\left(\frac{1}{2}\right)$, 0J2ΙΓK⇐RΓΙΓkMMΓ3∓FiI, Γ:Γ7ΤbNΙΓΙΓJΓCΓΙΓΗΓ8ΙΓ(ΓJΓΓ.

ΓˆΤ:, ΓΙΓΙΓˆΤGFb$\alpha(g,T,\theta)$ ΓO$\beta(T,\theta,\epsilon)$ ΓN$\epsilon>0$ Γ,=∓, Γ.Γ5ΤΙΓΗΓ-ΓN5sF0ΓrΓΙΓiΓoΓ9ΓΙΓaΓKF3FˆΤ7ΤΙNΙΓGFΙΓk Γ3ΓΗΓKCm0UΓ7ΤΗΓbΓiΓΙΓΙΓΓ. KΙMh,ΓdDΙNLT' −−−|−−T $\alpha h\alpha\alpha BgJP.\alpha\alpha N\alpha\alpha L\alpha\alpha\alpha N\alpha\alpha\alpha\alpha\alpha\alpha\alpha\alpha\alpha R\alpha J\alpha$<$\alpha\alpha D\alpha\alpha A\alpha\alpha\alpha\alpha k$ $\alpha\alpha\alpha\alpha\alpha\alpha\alpha k$: Γ9ΓJΓoΓA, GΙDUΓN$\theta\in\Theta,\epsilon>0$ ΓKΓDΓΙΓF$\beta(T',\theta,\epsilon)\ge\beta(T,\theta,\epsilon)$ ΓΗΓJΓkΓΗΓ-ΓKΓdDΙNLT' −−−|−−T $\alpha h\alpha\alpha BgJP.\alpha\alpha N\alpha\alpha L\alpha\alpha\alpha\alpha\alpha\alpha\alpha\alpha k\alpha\alpha\alpha\partial\alpha\alpha k$:

Γ7Τ∓F7ΤJΓ,Γi0l ΗΙΓKGΙDUΓN$\epsilon>0$ ΓKΓDΓΙΓΗΓN$\beta(T,\theta,\epsilon)$ ΓJΓΙΓNBgΓ.4Γ78Γ∓FiΓdDΙNΙΓNMΙΓ[Γr5DOΙΓ9ΓkΓ3ΓΗΓOJΓ ΓtΓJΓΙΓ,BΙΓΙ0l ΗΙΓKΓ6KΓaΓΗFqΓ7ΤΓ. Γ3ΓN5DOΙΓr4JC1ΓKΓ9ΓkΓΙΓaΓKF3FˆΤ5ΓlΓΙNΙΓ, $\alpha(g,T,\theta)$ ΓJΓΙΓGFΙΓk. Γ3Γl ΓiΓNNΙΓOΓhΓKOΓ5AΓ7ΤΓLD 7Γ6I=Γ6&JΙΓ-ΓrKΤΙΓ9ΓΗΓ-ΓOΓ3ΓlΓiΓNNΙΓNBgΓ.4Γ78Γ,0J2ΙΓN5DOΙΓr7PΓkΓ3ΓΗΓKΓhΓCΓF $\beta(T,\theta,\theta+\epsilon)$EyΓNNΙΓNBgΓ.4Γ78ΓK7kΓSΙUΓ1ΓkΓ3ΓΗΓ,2DG∓GFΙΓk.

Γ9ΓJΓoΓAΓΙΓNΓΙΓiΓaΓΙΓΙΓ, Θ ΓNΓ3ΓsΓΙΓΙΓΗt J,=8g Θ_0 Γ

,, ,, ,, ,, ,, (41) Γ,MΓ(Γk8B3&Γr:GLIΓdDINIΓOCIΓ.Γ7ΓEΓIΓk ΓˆΓΓ,:GLIΓdDINIΓOLCVΓK4IΓ7ΓE6&JIΓGFIΓkΓ+Fi, (71) ΓOθ $\alpha\alpha X\alpha\alpha F\alpha\alpha\alpha < dB\alpha\alpha\alpha F\alpha\alpha k$: Γ5ΓiΓK,ΓIΓKGLIΓdDINL$\hat{\theta}_{ML}$ $\beta r\, J\beta\beta A\beta\beta\beta\beta$ $\hat{\theta}'_{ML} := \{\hat{\theta}'_{ML,n}\}$ $--\in--------$ $--\|$.

$$\hat{\theta}'_{ML,n} := \begin{cases} 0 & \text{A4ΓEΓN}k\ \alpha\alpha I\alpha\alpha F X_k \in [-1,1] \\ \hat{\theta}_{ML,n} & \beta\beta\beta\beta\beta J\beta\beta \end{cases}.$$

$\alpha\alpha N\alpha\alpha\alpha\partial h\alpha\alpha\alpha I\alpha\alpha F\alpha\alpha\alpha h\alpha\alpha\alpha J\alpha\alpha$, $\alpha r\alpha\alpha\alpha k\alpha\alpha$, $g(x) = x^2$ Γ ʇ ʒ((

$(\alpha^+(\vec{a},g,T,\theta),\alpha^-(\vec{a},g,T,\theta))$ ΓNΓJΓ9=8g $\mathcal{C}(\vec{a},g,\theta)$ ΓJ(73) ΓGD5AΓ5ΓlΓkΓKΓKΓDΓIΓHΓIΓNDIMΓ,Γ.N)Γ9Γk

$$\mathcal{C}(\vec{a},g,\theta) := \{(\alpha^+(\vec{a},g,T,\theta),\alpha^-(\vec{a},g,T,\theta))|T: \quad \vec{a}\text{-6I=I6\&JIΓdDINL}\}. \tag{73}$$

DIMΓ4 (**(A.2)** ((

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,, ,j 16 5ZΓSAIBIPΓlΓKΓIΓ(ΓsΓHΓmΓTΓIΓNC1D4Γ-ΓhΓI0J2ΓNΓ0Γ,FIΓiΓlΓk

$$
\begin{aligned}
&-\lim_{n\to\infty} I^s(p_\theta^n\|p_{\theta+L/a_n}^n)\\
&\le \log\Bigg((P(\vec{a},T,\theta)(-\infty,0))^s\,(P(\vec{a},T,\theta)(-\infty,-L))^{1-s} + (P(\vec{a},T,\theta)(L,\infty))^s\,(P(\vec{a},T,\theta)(0,\infty))^{1-s}\\
&\qquad\qquad + \sum_{i=1}^{m}\left(P(\vec{a},T,\theta)\left(\frac{i-1}{m}L,\frac{i}{m}L\right)\right)^s\left(P(\vec{a},T,\theta)\left(\frac{i-1-m}{m}L,\frac{i-m}{m}L\right)\right)^{1-s}\Bigg)\\
&\le \log\left((P(\vec{a},T,\theta)(-\infty,-L))^{1-s} + (P(\vec{a},T,\theta)(L,\infty))^s + \sum_{i=1}^{m}\left(P(\vec{a},T,\theta)\left(\frac{i-1}{m}L,\infty\right)\right)^s\left(P(\vec{a},T,\theta)\left(-\infty,\frac{i-m}{m}L\right)\right)^{1-s}\right)\\
&\le \log\Bigg(\exp(1-s)\left(-\alpha^-(\vec{a},g,T,\theta)g(L)+\epsilon\right) + \exp s\left(-\alpha^+(\vec{a},g,T,\theta)g(L)+\epsilon\right)\\
&\qquad\qquad + \sum_{i=1}^{m}\exp\left(s\left(-\alpha^+(\vec{a},g,T,\theta)g\left(\frac{i-1}{m}L\right)+\epsilon\right)+(1-s)\left(-\alpha^-(\vec{a},g,T,\theta)g\left(\frac{m-i}{m}L\right)+\epsilon\right)\right)\Bigg)\\
&\le \log\left((m+2)\exp\left(-\min_{1\le i\le m} s\alpha^+(\vec{a},g,T,\theta)g\left(\frac{i-1}{m}L\right)+(1-s)\alpha^-(\vec{a},g,T,\theta)g\left(\frac{m-i}{m}L\right)+\epsilon\right)\right)\\
&= \log(m+2)-\left(\min_{1\le i\le m} s\alpha^+(\vec{a},g,T,\theta)g\left(\frac{i-1}{m}L\right)+(1-s)\alpha^-(\vec{a},g,T,\theta)g\left(\frac{m-i}{m}L\right)\right)+\epsilon.
\end{aligned}
$$

ααiααNϡααr $g(L)$ ΓG3dΓCΓIΓeΓG$L \to \infty$ −̸ ∀−−−−−‖−−∈−−−−−−‖.

$$
\lim_{L\to\infty}\frac{\lim\limits_{n\to\infty} I^s(p_\theta^n\|p_{\theta+L/a_n}^n)}{g(L)} \ge \lim_{L\to\infty}\frac{\min\limits_{1\le i\le m} s\alpha^+(\vec{a},g,T,\theta)g\left(\frac{i-1}{m}L\right)+(1-s)\alpha^-(\vec{a},g,T,\theta)g\left(\frac{m-i}{m}L\right)-\epsilon}{g(L)}.
$$

αααααα$\epsilon > 0$ ΓGGI0UΓGFIΓkΓ+Fi0J2ΓNΓ0ΓrFIΓk

$$
\lim_{L\to\infty}\frac{\lim\limits_{n\to\infty} I^s(p_\theta^n\|p_{\theta+L/a_n}^n)}{g(L)} \ge \lim_{L\to\infty}\frac{\min\limits_{1\le i\le m} s\alpha^+(\vec{a},g,T,\theta)g\left(\frac{i-1}{m}L\right)+(1-s)\alpha^-(\vec{a},g,T,\theta)g\left(\frac{m-i}{m}L\right)}{g(L)}.
$$

ααiαα, m αααααααααkαααααf αNαBαhααJα<αNαrF∂αk:

$$
\lim_{L\to\infty}\frac{\lim\limits_{n\to\infty} I^s(p_\theta^n\|p_{\theta+L/a_n}^n)}{g(L)} \ge \lim_{L\to\infty}\frac{\inf\limits_{0\le\eta\le 1} s\alpha^+(\vec{a},g,T,\theta)g(\eta L)+(1-s)\alpha^-(\vec{a},g,T,\theta)g((1-\eta)L)}{g(L)}.
$$

ααiααα$\alpha^-(\vec{a},g,T,\theta)=t\alpha^+(\vec{a},g,T,\theta)$ ΓGFIΓkΓ+Fi, 0J2ΓNΓ0ΓrFIΓk

$$
\lim_{L\to\infty}\frac{\lim\limits_{n\to\infty} I^s(p_\theta^n\|p_{\theta+L/a_n}^n)}{g(L)} \ge \alpha^+(\vec{a},g,T,\theta)\lim_{L\to\infty}\frac{\inf\limits_{0\le\eta\le 1} sg(\eta L)+(1-s)tg((1-\eta)L)}{g(L)}.
$$

αααααααCαF, $0 < s < 1$ Γ,GI0UΓGFIΓkΓ3ΓHΓ+Fi (74) ΓrFIΓk. □

8 ΓNΓZLIΓHF1MMKΓ7TF0J2ΓNJdBIΓrFIΓk.

JdBΓ**17** (**(A.2)** ((, *E,E*ﬃﬁﬃﬁﬃﬁﬃﬁﬀﬁﬃ [illegible] $\sigma_+(\vec{q}, \vartheta, T, \theta) = \sigma_-$

[illegible] $\sigma_+(\vec{q}, \vartheta, T, \theta)$ [illegible] $\sigma_-(\vec{q}, \vartheta, T, \theta)$ [illegible]

(A.2)

□

ΓΓ, $\vec{a} = \{n^{1/\gamma_1}\}, g(x) = |x|^{\gamma_1}$ ((((((. FΓK $\gamma_1 = \gamma_2 = 1$ ΓNΓH-ΓKΓDΓΓHΓOΓdDΓNΓ

GΓΓUΓNEΓΓG $\vec{a}$-6I=Γ6&JΓΓdDΓNΓΓGΓΓk Γ3ΓH-Fi, (78) Γ,E,MΓGF-0J2ΓΓNΓ0ΓrFΓΓk

,, ,j **19** $0 < \gamma_1 = \gamma_2 < 2$, $a_n = n^{\frac{1}{\gamma_1}}$ ((((((. Γ3ΓNΓH-, 0J2ΓΓNΓ0ΓrFΓΓk

$$\alpha^+(\vec{a}, g, \check{\theta}(\lambda), \theta) = A_1\left(\frac{1-\lambda}{\lambda} + 1\right), \quad \alpha^-(\vec{a}, g, \check{\theta}(\lambda), \theta) = A_2\left(\frac{\lambda}{1-\lambda} + 1\right).$$

$$P(\vec{a}, T, \theta)(-\infty, -L) \leq e^{-A_2 L^{\gamma_1}} \tag{81}$$

$$P(\vec{a}, T, \theta)(L, \infty) \leq e^{-A_1 L^{\gamma_1}} \tag{82}$$

$\alpha\alpha\alpha\alpha\alpha\alpha C\alpha F$ $\gamma_1 = \gamma_2 = 1$ ΓNΓH-ΓKΓDΓΓHFO Order ΓΓΓ1ΓGFOΓJΓΓ.G9bΓΓΓN78ΓtΓN0ULΓΓG8B3&ΓrCΓΓ.Γ7ΓHΓ

ΓZLΓ (57) ΓhΓΓ, ΓN5DOΓΓ-FiΓr7o **(A.4)** **(** **9**

$$P(\vec{a}, T, \theta)(-\infty, -L) \leq \lim_{n\to\infty}\left(1 - \int_{b-\frac{L}{(1-\lambda)n^{\frac{1}{\gamma_1}}}}^{b} f(x)\,dx\right)^n = e^{-A_2 L^{\gamma_1}} \tag{83}$$

ΓHΓJΓΓ(81) ΓrFΓΓk. F1MΓMK(82) ΓrFΓΓk. □

□ □

$$= \lim_{L\to\infty} \frac{1}{g(L)} \sup_{0<s<1} \lim_{n\to\infty} I^s(F_\theta \| F_{\theta+\frac{2L}{a_n}}). \qquad (86)$$

$$\lim_{L\to\infty} \frac{1}{g(L)} \sup \{-\log \max \{P(\vec{a}, T, \theta)(L, \infty), P(\vec{a}, T, \theta)(-\infty, -L)\} \mid T: 0L$$ [illegible]

□ $L \to +\infty$ [illegible] (61) [illegible]

,, ,, ,k . s [illegible] sup [illegible]

$$-\log P(\vec{a}, \tilde{\theta}^L, \theta)(L, \infty) \geq \sup_{0<s<1} \lim_{n\to\infty} I^s(F_\theta \| F_{\theta+\frac{2L}{a_n}})$$

[illegible]

$$-\log P(\vec{a}, \tilde{\theta}^L, \theta)(-\infty, -L) \geq \sup_{0<s<1} \lim_{n\to\infty} I^s(F_\theta \| F_{\theta+\frac{2L}{a_n}})$$

[illegible] (84) [illegible] □

□□□ (

$$\sup\left\{\lim_{r\to+\infty}\frac{-1}{r^2}\log\left(\max\{P(q,T,\theta)(r,\infty),P(q,T,\theta)(-\infty,-r)\}\right)\,\middle|\,T:\ [illegible]\right\}=\frac{[illegible]}{A_1+A_2}. \quad (90)$$

$$\lim_{r\to+\infty}\frac{1}{r^2}\sup\left\{-\log\left(\max\{P(q,T,\theta)(r,\infty),P(q,T,\theta)(-\infty,-r)\}\right)\,\middle|\,T:\ [illegible]\right\}=\frac{4}{A_1+A_2} \quad (89)$$

[illegible] $\{\sqrt{n\log n}\}$ [illegible]

$\lambda_1=\lambda_2=2$ [illegible] (87), (88) [illegible]

(A.5) $\geq$ [illegible] (90) [illegible]

[illegible] **(A.3)** ((**2**((

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A 2 1 11 1 (A.3) (

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,h ,, ,, (22) Γ,Γ(ΓΠΓΓ.

0J2ΠΓr7o **(A.5) (** ((((

„ „ $\gamma_1 = 1$ ΓΝΓΗΓ-ΓΝ$\tilde{I}_s^-(c,f,\epsilon)$ ΓΝ6Κ8ΒΓrD4ΓΙΓk (95) ΓrΜΓΙΓkΓΗ0J2ΓΙΓΝΓ0ΓrFΓΙΓk

$$\begin{aligned}
&\tilde{I}_s^-(c,f,\epsilon)\\
&\cong_{\epsilon^2\to+0} \int_a^c f(x)\,dx + \epsilon s f(c) + \frac{\epsilon^2}{2} s f'(c) - A_1 s\epsilon + \left(\frac{s(s-1)}{2}\int_a^c f^{-1}(x)(f'(x))^2\,dx\right)\epsilon^2. \qquad (100)
\end{aligned}$$

ΓΙΓΚ$1 < \gamma_1 \le 2$ ΓΝΓΗΓ-ΓΝ$\tilde{I}_s^-(c,f,\epsilon)$ ΓΝ6Κ8ΒΓrD4ΓΙΓk Γ3ΓΝΓΗΓ-0J2ΓΙΓΝΓ0ΓrFΓΙΓk

$$\begin{aligned}
&\tilde{I}_s^-(c,f,\epsilon)\\
&\cong_{\epsilon^2\to+0} \int_{a+\delta}^c \left(f(x) + f^{1-s}(x)(f^s)'(x)\epsilon + f^{1-s}(x)(f^s)''(x)\frac{\epsilon^2}{2}\right)dx + \int_a^{a+\delta} f^{1-s}(x) f^s(x+\delta)\,dx\\
&= \int_a^c \left(f(x) + f'(x)s\epsilon\right)dx + \frac{s(s-1)}{2}\left(\int_{a+\delta}^c f^{-1}(x)(f'(x))^2\,dx\right)\epsilon^2 + \frac{s}{2}\left(\int_{a+\delta}^c f''(x)\,dx\right)\epsilon^2\\
&\quad + \int_a^{a+\delta} f^{1-s}(x) f^s(x) - f(x) - \epsilon f^{1-s}(x)(f^s)'(x)\,dx\\
&= \int_a^c f(x)\,dx + (f(c) - f(a))s\epsilon + (f'(c) - f'(a+\delta))s\frac{\epsilon^2}{2}\\
&\quad + \frac{s(1-s)}{2}\left(\int_{a+\delta}^c f^{-1}(x)(f'(x))^2\,dx\right)\epsilon^2 + \int_a^{a+\delta}\int_0^\epsilon\int_0^{y_1} f^{1-s}(x)(f^s)''(x+y_2)\,dy_2\,dy_1\,dx. \qquad (101)
\end{aligned}$$

(101) ΓΝG=ΒΓΙΓΟ$1 < \gamma_1 < 2$ ΓΝΓΗΓ-ΓΟΓ5ΓiΓΚ$z = \frac{x}{y_2}$ ((((((((0(2(((((((((((((((((((.

$$\begin{aligned}
&\frac{1}{A_1 s(\gamma_1 - 1)(s(\gamma_1 - 1) - 1)}\int_a^{a+\delta}\int_0^\epsilon\int_0^{y_1} f^{1-s}(x)(f^s)''(x+y_2)\,dy_2\,dy_1\,dx\\
&= \int_0^\delta\int_0^\epsilon\int_0^{y_1} x^{(1-s)(\gamma_1-1)}(x+y_2)^{s(\gamma_1-1)-2}\,dy_2\,dy_1\,dx\\
&= \int_0^\epsilon\int_0^{y_1}\int_0^\delta x^{(1-s)(\gamma_1-1)}(x+y_2)^{s(\gamma_1-1)-2}\,dx\,dy_2\,dy_1\\
&= \int_0^\epsilon\int_0^{y_1}\int_0^{\frac{\delta}{y_2}} z^{(1-s)(\gamma_1-1)}(z+1)^{s(\gamma_1-1)-2} y_2^{\gamma_1-2}\,dz\,dy_2\,dy_1\\
&= \int_0^\epsilon\int_0^{y_1}\left(B(2-\gamma_1, (1-s)(\gamma_1-1)+1) - \int_{\frac{\delta}{y_2}}^\infty z^{(1-s)(\gamma_1-1)}(z+1)^{s(\gamma_1-1)-2}\,dz\right) y_2^{\gamma_1-2}\,dy_2\,dy_1\\
&= B(2-\gamma_1, (1-s)(\gamma_1-1)+1)\frac{1}{\gamma_1(\gamma_1-1)}\epsilon^\gamma - \int_0^\epsilon\int_0^{y_1}\int_{\frac{\delta}{y_2}}^\infty z^{\gamma_1-3}\left(1+\frac{1}{z}\right)^{s(\gamma_1-1)-2} dz y_2^{\gamma_1-2}\,dy_2\,dy_1\\
&\cong_{\epsilon^2\to0} B(2-\gamma_1, (1-s)(\gamma_1-1)+1)\frac{1}{\gamma_1(\gamma_1-1)}\epsilon^\gamma - \int_0^\epsilon\int_0^{y_1}\int_{\frac{\delta}{y_2}}^\infty z^{\gamma_1-3}\,dz y_2^{\gamma_1-2}\,dy_2\,dy_1\\
&= B(2-\gamma_1, (1-s)(\gamma_1-1)+1)\frac{1}{\gamma_1(\gamma_1-1)}\epsilon^\gamma - \frac{1}{2(\gamma_1-2)}\delta^{\gamma_1-2}\epsilon^2. \qquad (102)
\end{aligned}$$

Γ7ΓΙΓ,ΓΟΓF(101) ΓΗ(102) ΓrAΗΓΙ9gΓoΓΙΓkΓΗ$1 < \gamma_1 < 2$ ΓΝΓΗΓ-ΓΚΓDΓΙΓΗΓΟ0J2ΓΙΓΝΓ0ΓrFΓΙΓk

$$\begin{aligned}
&\tilde{I}_s^-(c,f,\epsilon)\\
&\cong_{\epsilon^2\to+0} \int_a^c f(x)\,dx + f(c)s\epsilon + f'(c)\frac{s}{2}\epsilon^2\\
&\quad + \frac{A_1 s(\gamma_1-1)(s(\gamma_1-1)-1)}{\gamma_1(\gamma_1-1)} B(2-\gamma_1, (1-s)(\gamma_1-1)+1)\epsilon^\gamma\\
&\quad + \left(\frac{s(s-1)}{2}\left(\int_{a+\delta}^c f^{-1}(x)(f'(x))^2\,dx + \frac{A_1(\gamma_1-1)^2}{\gamma_1-2}\delta^{\gamma_1-2}\right)\right)\epsilon^2. \qquad (103)
\end{aligned}$$

$\gamma_1 = 2$ ΓNΓH-ΓKΓDΓIΓHΓO(101) ΓNG=I9ΓIΓO $z = \frac{x}{y_2}$ ((((((((0(2(((((((((((((((((((.

$$\begin{aligned}
&\frac{1}{A_1 s(s-1)} \int_a^{a+\delta} \int_0^{\epsilon} \int_0^{y_1} f^{1-s}(x)(f^s)''(x+y_2)\, dy_2\, dy_1\, dx \\
&= \int_0^{\delta} \int_0^{\epsilon} \int_0^{y_1} x^{1-s}(x+y_2)^{s-2}\, dy_2\, dy_1\, dx \\
&= \int_0^{\epsilon} \int_0^{y_1} \int_0^{\delta} x^{1-s}(x+y_2)^{s-2}\, dx\, dy_2\, dy_1 \\
&= \int_0^{\epsilon} \int_0^{y_1} \int_0^{\frac{\delta}{y_2}} z^{1-s}(z+1)^{s-2}\, dz\, dy_2\, dy_1. \qquad (104)
\end{aligned}$$

0l JΓ, $v = \frac{z}{1+x}$ $\beta\beta\beta\beta\beta\beta\beta\beta\beta J\beta<\beta\beta<\beta\beta\beta\partial\beta\beta\beta\beta\beta\beta\beta$.

$$\lim_{y_2 \to 0} \int_0^{\frac{\delta}{y_2}} \left(z^{1-s}(z+1)^{s-2} - \frac{1}{1+z} \right) dz = \int_0^1 \frac{1 - v^{1-s}}{1-v}\, dv = -E_u - \psi(2-s) \qquad (105)$$

$$\lim_{y_2 \to 0} \int_0^{\frac{\delta}{y_2}} \frac{1}{1+z}\, dz - (\log \delta - \log y_2) = 0.$$

(105) ΓNF3=IPΓKΓDΓIΓHΓONdΓ(ΓHΓ98Γ, 1' EDΓn, 0l ΓΓ[19] ΓNp.225 ΓrΓ2ΓHΓNΓ3ΓH Γ7ΓIΓ,ΓCΓF

$$\begin{aligned}
&\frac{1}{A_1 s(s-1)} \int_a^{a+\delta} \int_0^{\epsilon} \int_0^{y_1} f^{1-s}(x)(f^s)''(x+y_2)\, dy_2\, dy_1\, dx \\
&\cong_{\epsilon^2 \to 0} \int_0^{\epsilon} \int_0^{y_1} \log \delta - \log y_2 - E_u - \psi(2-s)\, dy_2\, dy_1 \\
&= -\frac{1}{2}\epsilon^2 \log \epsilon + \frac{1}{2} \left(\log \delta + \frac{3}{2} - E_u - \psi(2-s) \right) \epsilon^2 \qquad (106)
\end{aligned}$$

ΓHΓJΓk $\gamma_1 = 2$ ΓNΓH-ΓKΓDΓIΓO(101) ΓH(106) ΓrAHΓI9gΓoΓIΓkΓH0J2ΓIΓNΓ0ΓrFΓIΓk

$$\begin{aligned}
&\tilde{I}_s^-(c, f, \epsilon) \\
&\cong_{\epsilon^2 \to +0} \int_a^c f(x)\, dx + f(c)s\epsilon + f'(c)\frac{s}{2}\epsilon^2 + \frac{A_1 s(1-s)}{2}\epsilon^2 \log \epsilon \\
&\quad + \frac{s(s-1)}{2} \left(\int_{a+\delta}^c f^{-1}(x)(f'(x))^2\, dx + A_1 \log \delta \right) \epsilon^2 + \frac{A_1 s}{2} \left(-1 + (1-s) \left(-\frac{3}{2}(1-s) + E_u + \psi(2-s) \right) \right) \epsilon^2 \\
&\hfill (107)
\end{aligned}$$

$\gamma > 2$ ΓNΓH-ΓOΓ0 (93) Γ,E,MΓGF-ΓF(23) Γ,Γ=ΓNΓ^Γ^Γ(ΓIΓk
ΓIΓK(24) ΓrΓ(Γ9ΓIΓaΓK $\tilde{I}_s^+(c, f, \epsilon)$ ΓN6K8BΓrD4ΓIΓk ΓOΓ8ΓaΓK0J2ΓIΓNΓ0Γ,Γ.N)Γ9Γk

$$\begin{aligned}
&\tilde{I}_s^+(c, f, \epsilon) \\
&\cong_{\epsilon^2 \to +0} \int_c^{b-\delta} f(x) + \epsilon f^{1-s}(x)(f^s)'(x) + \frac{\epsilon^2}{2} f^{1-s}(x)(f^s)''(x)\, dx + \int_{b-\delta}^{b-\epsilon} f^{1-s}(x) f^s(x+\epsilon)\, dx. \qquad (108)
\end{aligned}$$

$0,\ 2,\ \ 0 < \gamma_2 < 1$ ΓNΓH-ΓKΓDΓIΓF$\tilde{I}_s^+(c,f,\epsilon)$ΓN6K8BΓrD4ΓIΓk

$$\begin{aligned}
&\tilde{I}_s^+(c,f,\epsilon) \\
&\cong_{\epsilon^2\to+0} \int_{b-\delta+\epsilon}^{b} \left(f^{1-s}(x-\epsilon)f^s(x)-f(x)\right)\,dx + \int_c^b f(x)\,dx - \int_{b-\delta}^{b-\delta+\epsilon} f(x)\,dx \\
&\quad + \left(\int_c^{b-\delta} f^{1-s}(x)(f^s)'(x)\,dx\right)\epsilon + \left(\int_c^{b-\delta} f^{1-s}(x)(f^s)''(x)\,dx\right)\frac{\epsilon^2}{2} \\
&= \int_{b-\delta+\epsilon}^{b} f^s(x)\int_{-\epsilon}^{0} (f^{1-s})'(x+y)\,dy\,dx + \int_c^b f(x)\,dx - \int_{b-\delta}^{b-\delta+\epsilon} f(x)\,dx \\
&\quad + \left(\int_c^{b-\delta} f'(x)\,dx\right)s\epsilon + \left(\frac{s(s-1)}{2}\int_c^{b-\delta}(s-1)f^{-1}(x)(f'(x))^2\,dx + \frac{s}{2}\int_c^{b-\delta} f''(x)\,dx\right)\epsilon^2 \\
&= \int_{b-\delta+\epsilon}^{b} f^s(x)\int_{-\epsilon}^{0} (f^{1-s})'(x+y)\,dy\,dx + \int_c^b f(x)\,dx - \int_{b-\delta}^{b-\delta+\epsilon} f(x)\,dx \\
&\quad + (f(b-\delta)-f(c))s\epsilon + (f'(b-\delta)-f'(c))\frac{s}{2}\epsilon^2 + \frac{s(s-1)}{2}\left(\int_c^{b-\delta} f^{-1}(x)(f'(x))^2\,dx\right)\epsilon^2.
\end{aligned} \tag{109}$$

Γ=F3ΓG1&JUΓNBh1 9ΓIΓKΓDΓIΓEΓO$z = \frac{x}{y}$ ββββββkβββJβ<ββ<ββrβ∂βk.

$$\begin{aligned}
&\frac{1}{A_2(1-s)(\gamma_2-1)}\int_{b-\delta+\epsilon}^{b} f^s(x)\int_{-\epsilon}^{0}(f^{1-s})'(x+y)\,dy\,dx \\
&= \int_{-\delta+\epsilon}^{0}\int_{-\epsilon}^{0}\frac{(-x)^{s(\gamma_2-1)}}{(-x-y)^{s(\gamma_2-1)+2-\gamma_2}}\,dy\,dx \\
&= \int_{-\epsilon}^{0}\int_{-\delta+\epsilon}^{0}\frac{(-x)^{s(\gamma_2-1)}}{(-x-y)^{s(\gamma_2-1)+2-\gamma_2}}\,dx\,dy \\
&= \int_{0}^{\epsilon}\int_{0}^{\delta-\epsilon}\frac{x^{s(\gamma_2-1)}}{(x+y)^{s(\gamma_2-1)+2-\gamma_2}}\,dx\,dy \\
&= \int_{0}^{\epsilon}\int_{0}^{\frac{\delta-\epsilon}{y}}\frac{z^{s(\gamma_2-1)}}{(z+1)^{s(\gamma_2-1)+2-\gamma_2}}y^{\gamma_2-1}\,dz\,dy \\
&= \int_{0}^{\epsilon}\left(B(s(\gamma_2-1)+1,1-\gamma_2) - \int_{\frac{\delta-\epsilon}{y}}^{\infty}\frac{z^{s(\gamma_2-1)}}{(z+1)^{s(\gamma_2-1)+2-\gamma_2}}\,dz\right)y^{\gamma_2-1}\,dy \\
&= \int_{0}^{\epsilon}\left(B(s(\gamma_2-1)+1,1-\gamma_2) - \int_{\frac{\delta-\epsilon}{y}}^{\infty} z^{\gamma_2-2}\left(1+\frac{1}{z}\right)^{-s(\gamma_2-1)+\gamma_2-2}\,dz\right)y^{\gamma_2-1}\,dy \\
&\cong_{\epsilon^2\to0}\int_{0}^{\epsilon}\left(B(s(\gamma_2-1)+1,1-\gamma_2) - \int_{\frac{\delta-\epsilon}{y}}^{\infty} z^{\gamma_2-2} + (-s(\gamma_2-1)+\gamma_2-2)z^{\gamma_2-3}\,dz\right)y^{\gamma_2-1}\,dy \\
&= B(s(\gamma_2-1)+1,1-\gamma_2)\frac{1}{\gamma_2}\epsilon^{\gamma_2} + \frac{(\delta-\epsilon)^{\gamma_2-1}}{\gamma_2-1}\epsilon + \frac{(-s(\gamma_2-1)+\gamma_2-2)(\delta-\epsilon)^{\gamma_2-2}}{2(\gamma_2-2)}\epsilon^2 \\
&\cong_{\epsilon^2\to0} B(s(\gamma_2-1)+1,1-\gamma_2)\frac{1}{\gamma_2}\epsilon^{\gamma_2} + \frac{\delta^{\gamma_2-1}}{\gamma_2-1}\epsilon + \left(\frac{(-s(\gamma_2-1)+\gamma_2-2)\delta^{\gamma_2-2}}{2(\gamma_2-2)} - \delta^{\gamma_2-2}\right)\epsilon^2 \\
&= B(s(\gamma_2-1)+1,1-\gamma_2)\frac{1}{\gamma_2}\epsilon^{\gamma_2} + \frac{\delta^{\gamma_2-1}}{\gamma_2-1}\epsilon + \left(-\frac{s(\gamma_2-1)\delta^{\gamma_2-2}}{2(\gamma_2-2)} - \frac{1}{2}\delta^{\gamma_2-2}\right)\epsilon^2.
\end{aligned} \tag{110}$$

,, ,, ,, ,, 0, 2, ,, ,0 ,, @.,, ,9 ,k .

$$\int_{b-\delta}^{b-\delta+\epsilon} f(x)\,dx \cong_{\epsilon^2\to 0} A_2\left(\delta^{\gamma_2-1}\epsilon + \frac{\gamma_2-1}{2}\delta^{\gamma_2-2}\epsilon^2\right) \tag{111}$$

$$f(b-\delta)s\epsilon = A_2\delta^{\gamma_2-1} \tag{112}$$

$$f'(b-\delta)\frac{s}{2}\epsilon^2 = -A_2(\gamma_2-1)\delta^{\gamma_2-2}. \tag{113}$$

(109) ΓK(110),(111),(112) 5ZΓS(113) ΓrAHΓI9gΓoΓIΓkΓH0J2ΓΓNΓ0ΓrFIΓk

$$\begin{aligned}
&\tilde{I}_s^+(c,f,\epsilon)\\
&\cong_{\epsilon^2\to+0} \int_c^b f(x)\,dx - f(c)s\epsilon - f'(c)\frac{s}{2}\epsilon^2\\
&\quad + B(s(\gamma_2-1)+1, 1-\gamma_2)\frac{A_2(1-s)(\gamma_2-1)}{\gamma_2}\epsilon^{\gamma_2} + \left(\int_c^{b-\delta} f^{-1}(x)(f'(x))^2\,dx + \frac{A_2(\gamma_2-1)^2}{\gamma_2-2}\delta^{\gamma_2-1}\right)\frac{s(s-1)}{2}\epsilon^2.
\end{aligned} \tag{114}$$

ΓIΓKγ₂ = 1 ΓNΓH-ΓKΓDΓIΓF$\tilde{I}_s^+(c,f,\epsilon)$ ΓN6K8BΓrD4ΓIΓk (96) ΓrMΓIΓkΓH0J2ΓΓNΓ0ΓrFIΓk

$$\begin{aligned}
&\tilde{I}_s^+(c,f,\epsilon)\\
&\cong_{\epsilon^2\to 0} \int_c^b f(x)\,dx - f(c)s\epsilon - f'(c)\frac{s}{2}\epsilon^2 + A_2(s-1)\epsilon + \left(\int_c^b f^{-1}(x)(f')^2(x)\,dx\right)\frac{s(s-1)}{2}\epsilon^2.
\end{aligned} \tag{115}$$

,, ,, $1 < \gamma_2 \leq 2$ ΓNΓH-ΓKΓDΓIΓΓF$\tilde{I}_s^+(c,f,\epsilon)$ ΓN6K8BΓrD4ΓIΓk (108) ΓrΓ29MΓKΓ9ΓkΓH0J2ΓΓNΓ0Γ,Γ.ΓIN)ΓD

$$
\begin{aligned}
&\tilde{I}_s^+(c,f,\epsilon)\\
&\cong_{\epsilon^2\to 0} \int_c^b \left(f(x)+f'(x)s\epsilon\right)\,dx - \int_{b-\delta}^b \left(f(x)+f'(x)s\epsilon\right)\,dx + \left(\int_c^{b-\delta} f^{1-s}(x)(f^s)''(x)\,dx\right)\frac{\epsilon^2}{2}\\
&\quad + \int_{b-\delta+\epsilon}^b f^s(x)\left(f^{1-s}(x-\epsilon) - f^{1-s}(x) + (f^{1-s})'(x)\epsilon\right)\,dx + \int_{b-\delta+\epsilon}^b \left(f(x) - f^s(x)(f^{1-s})'(x)\epsilon\right)\,dx\\
&= \int_c^b f(x)\,dx + \left(\int_c^b f'(x)\,dx\right)s\epsilon + \left(\int_c^{b-\delta} \frac{s(s-1)}{2}f^{-1}(x)(f'(x))^2 + \frac{s}{2}f''(x)\,dx\right)\epsilon^2\\
&\quad + \int_{b-\delta+\epsilon}^b f^s(x)\left(-\int_{-\epsilon}^0 (f^{1-s})'(x+y_1) - f^{1-s}(x)\,dy_1\right)\,dx\\
&\quad + \int_{b-\delta+\epsilon}^{b-\delta} f(x)\,dx - \left(\int_{b-\delta}^b f'(x)\,dx\right)s\epsilon + \left(\int_{b-\delta+\epsilon}^b f'(x)\,dx\right)(s-1)\epsilon\\
&\cong_{\epsilon^2\to 0} \int_c^b f(x)\,dx + (f(b)-f(c))s\epsilon + \left(\int_c^{b-\delta} \frac{s(s-1)}{2}f^{-1}(x)(f'(x))^2\,dx + \frac{s}{2}f'(b-\delta)\right)\epsilon^2\\
&\quad + \int_{b-\delta+\epsilon}^b f^s(x)\left(-\int_{-\epsilon}^0\int_0^{y_1} (f^{1-s})''(x+y_2)\,dy_2\,dy_1\right)\,dx\\
&\quad - \int_{b-\delta}^{b-\delta+\epsilon} \left(f(b-\delta) + f'(b-\delta)(x-(b-\delta))\right)\,dx - \left(\int_{b-\delta}^{b-\delta+\epsilon} f'(x)\,dx\right)s\epsilon - \left(\int_{b-\delta+\epsilon}^b f'(x)\,dx\right)\epsilon\\
&\cong_{\epsilon^2\to 0} \int_c^b f(x)\,dx - f(c)s\epsilon - f'(c)\frac{s}{2}\epsilon + f(b)s\epsilon + \left(\int_c^{b-\delta} \frac{s(s-1)}{2}f^{-1}(x)(f'(x))^2\,dx + \frac{s}{2}f'(b-\delta)\right)\epsilon^2\\
&\quad + \int_{b-\delta+\epsilon}^b f^s(x)\left(-\int_{-\epsilon}^0\int_0^{y_1} (f^{1-s})''(x+y_2)\,dy_2\,dy_1\right)\,dx\\
&\quad - \left(f(b-\delta)\epsilon + f'(b-\delta)\frac{\epsilon^2}{2}\right) - (f(b-\delta+\epsilon) - f(b-\delta))s\epsilon - (f(b) - f(b-\delta+\epsilon))\epsilon\\
&\cong_{\epsilon^2\to 0} \int_c^b f(x)\,dx - f(c)s\epsilon - f'(c)\frac{s}{2}\epsilon^2 + f(b)s\epsilon + \left(\int_c^{b-\delta} \frac{s(s-1)}{2}f^{-1}(x)(f'(x))^2\,dx + \frac{s}{2}f'(b-\delta)\right)\epsilon^2\\
&\quad - \int_{b-\delta+\epsilon}^b f^s(x)\int_{-\epsilon}^0\int_0^{y_1} (f^{1-s})''(x+y_2)\,dy_2\,dy_1\,dx\\
&\quad - \left(f(b-\delta)\epsilon + f'(b-\delta)\frac{\epsilon^2}{2}\right) - f'(b-\delta)s\epsilon^2 - (f(b) - f(b-\delta) - f'(b-\delta)\epsilon)\epsilon\\
&= \int_c^b f(x)\,dx - f(c)s\epsilon - f'(c)\frac{s}{2}\epsilon^2 + \left(\int_c^{b-\delta} \frac{s(s-1)}{2}f^{-1}(x)(f'(x))^2\,dx\right)\epsilon^2\\
&\quad - \int_{b-\delta+\epsilon}^b f^s(x)\int_{-\epsilon}^0\int_0^{y_1} (f^{1-s})''(x+y_2)\,dy_2\,dy_1\,dx - f(b)(1-s)\epsilon + f'(b-\delta)\frac{1-s}{2}\epsilon^2. \qquad (116)
\end{aligned}
$$

$$\frac{1}{A_2(1-s)(\gamma_2-1)((1-s)(\gamma_2-1)-}$$

$$\le \int_0^{\delta}\int_0^{\epsilon}\int_0^{y_1} \frac{x^{s(\gamma_2-1)}}{(x+y_2)^{2-(1-s)(\gamma_2-1)}}$$

,, ,, ,, $1 < \gamma_2 < 2$ ΓNΓH-ΓKΓO(102) ΓrΓ29MΓKΓ9ΓkΓH$z = \frac{x}{y_2}$ ((((((((0(2((((0 (

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