(as of February 5th)

Titles & Abstracts

Analytical aspects of the $\overline{\partial}$ equation
Graduate School of Mathematics, Nagoya University, Room 309
February 13–15, 2015

Masanori Adachi

Curvature restrictions for Levi-flat real hypersurfaces in complex projective planes

We study curvature restrictions of Levi-flat real hypersurfaces in complex projective planes, whose existence is in question. We focus on its totally real Ricci curvature, the Ricci curvature of the real hypersurface in the direction of the Reeb vector field, and show that it cannot be greater than -4 along a Levi-flat real hypersurface. We rely on a finiteness theorem for the space of square integrable holomorphic 2-forms on the complement of the Levi-flat real hypersurface, where the curvature plays the role of the size of the infinitesimal holonomy of its Levi foliation. This is a joint work with Judith Brinkschulte.

John Erik Fornæss

Squeezing functions

This is joint work with Erlend F. Wold. We discuss the squeezing function on strongly pseudoconvex domains in $\mathbb{C}^n$. The question is how the squeezing function behaves near the boundary.

Anne-Katrin Herbig

On the closed range property of $\overline{\partial}$ Abstract: A potential-theoretic condition sufficient for the range of $\overline{\partial}$ to be closed on pseudoconvex, unbounded domains is discussed. The talk is based on joint work with Jeff McNeal.

Pham Hoang Hiep

Recent development about the strong openness conjecture and effective version of the semicontinuity theorem

In this talk, we show how to apply the original L^2-extension theorem of Ohsawa and Takegoshi to the standard basis of a multiplier ideal sheaf associated with a plurisubharmonic function. In this way, we are able to reprove the strong openness conjecture and to obtain an effective version of the semicontinuity theorem for weighted log canonical thresholds.

Chin-Yu Hsiao

Szegö kernel asymptotics and Kodaira embedding theorems on Levi-flat CR manifolds

Let L be a positive CR complex line bundle over a compact Levi-flat CR manifold X. In this work, we prove that a certain microlocal conjugation of the associated Szegö kernel admits an asymptotic expansion with respect to high powers of L. As an application, we give a Szegö kernel proof of Ohsawa and Sibony's Kodaira type embedding theorems on Levi-flat CR manifolds.

Hideyuki Ishi

CR Laplacian-type operator on the boundary of the generalized upper half plane

We discuss analysis on the boundary of the generalized upper half plane, which is the Siegel domain realization of the unit ball. It is known that the boundary is naturally identified with the Heisenberg Lie group, where the CR structure is described efficiently in a Lie theoretical way. In this talk, we introduce a special inner product on a space of forms so that the associated Laplacian-type operator has discrete spectrums.

Takayuki Koike

Toward a higher codimensional Ueda theory

Ueda's theory is a theory on a flatness criterion around a smooth hypersurface of a certain type of topologically trivial holomorphic line bundles. We propose a codimension two analogue of Ueda's theory. As an application, we give a sufficient condition for the anti-canonical bundle of the blow-up of the three dimensional projective space at 8 points to be non semi-ample however admit a smooth Hermitian metric with semi-positive curvature.

Xiaoshan Li

Szegö kernel asymptotics and Morse inequalities on CR manifolds with S^1 action

Let X be a compact orientable CR manifold of dimension $(2n-1)$, $n \geq 2$. We assume there is a transversal CR S^1 action on X. Let L^k be the k-th tensor power of a rigid CR line bundle L over X. Without any assumption on the Levi-form of X, we obtain a scaling upper-bound for the Szegö kernel on $(0,q)$-forms with values in L^k, for large k which generalize the results of Hsiao and Marinescu. After integration, this gives the weak Morse inequalities. From the weak Morse inequalities in our setting, the holomorphic Morse inequalities of Demailly is derived. By a refined spectral analysis, we obtain also strong Morse inequalities. We apply the strong Morse inequalities to show that the Grauert-Riemenschneider conjecture is also true in the CR setting. This is a joint work with Professor Hsiao, Chin-Yu.

Shin-ichi Matsumura

Versions of injectivity theorems

In this talk, I will explain some generalizations of Kollar's injectivity theorem, which can be seen as a generalization of the cohomology vanishing theorem. In particular, I would like to discuss relations between the proof of injectivity theorem and L^2-esimates of solutions of the dbar-equation

Takeo Ohsawa

Compactly supported cohomology by the L^2 method

It is known that, for any complex manifold M and for any holomorphic vector bundle E over M, $H_c^{0,1}(M,E)$, the compactly supported E-valued $\overline{\partial}$-cohomology group of type $(0,1)$, is Hausdorff. This is an immediate consequence of a combination of Siu's generalization of Malgrange's vanishing theorem and Serre duality. Similarly, $H_c^{0,n-q+1}(M,E)$ are Hausdorff if M is a connected n-dimensional q-convex manifold. In two papers written in 1980, I refined Andreotti-Grauert's finiteness theorem by extending Andreotti-Vesentini's L^2 theory and showed that the Hodge theory can be extended to a class of complete Kähler manifolds which are also q-convex. The result was about the de Rham cohomology of degree at least $n+q$, and applied recently to the study of Levi flat hypersurfaces. By the L^2 method, it will be shown that, for any n-dimensional q-convex bounded domain D with smooth boundary in a complex manifold M and for any holomorphic Hermitian vector bundle E over M, there exists a complete Hermitian metric say ω on D such that the natural homomorphism from $H_c^{0,p}(D,E)$ to the space of E-valued L^2 harmonic $(0,p)$-forms on D with respect to ω is bijective if $p < n-q+1$ and injective if $p = n-q+1$. As a result, besides the Hausdorffness, one can see a natural Hodge-type filtration in $H_c^1(M,C)$, if M is pseudoconvex and Kählerian.

Elizabeth Wulcan

Nonproper intersection theory and generalized cycles

I will discuss a joint work in progress with Mats Andersson, Dennis Eriksson, Håkan Samuelsson Kalm, and Alain Yger, which aims at giving an analytic approach to nonproper intersection theory.

Given two analytic cycles Z and W that intersect properly, there is a nice analytic interpretation of their intersection cycle as the product $[Z] \wedge [W]$ of the currents of integration along Z and W.

To deal with the nonproper case we introduce a class of currents that we call generalized cycles and that contains all analytic cycles. Each generalized cycle has a well-defined multiplicity at each point and a well-defined degree. The intersection of two (generalized) cycles Z and W is a generalized cycle $Z \bullet W$ whose degree satisfies Bezout's theorem. Moreover the multiplicities of $Z \bullet W$ are the local intersection numbers in the sense of Tworzewski, Achilles-Manaresi, and Gaffney-Gassler.

Po Lam Yung

A new algebra of pseudodifferential operators

In this talk, we will construct a class of pseudodifferential operators, that can be used to describe some naturally arising operators in several complex variables. The main geometric data needed in this construction is a distribution of hyperplanes, on say the boundary of a domain in $\mathbb{C}^N$. The pseudodifferential operators we construct can be used to understand what happens when one composes an operator with Euclidean homogeneity, with an operator with a Heisenberg homogeneity. This is joint work with E. Stein.

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