

Szegö kernel asymptotics and Morse inequalities on CR manifolds with S^1 action

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Abstract

Let X be a compact orientable CR manifold of dimension $(2n - 1), n \geq 2$. We assume there is a transversal CR S^1 action on X . Let L^k be the k -th tensor power of a rigid CR line bundle L over X . Without any assumption on the Levi-form of X , we obtain a scaling upper-bound for the Szegö kernel on $(0, q)$ -forms with values in L^k , for large k which generalize the results of Hsiao and Marinescu. After integration, this gives the weak Morse inequalities. From the weak Morse inequalities in our setting, the holomorphic Morse inequalities of Demailly is derived. By a refined spectral analysis, we obtain also strong Morse inequalities. We apply the strong Morse inequalities to show that the Grauert-Riemenschneider conjecture is also true in the CR setting. This is a joint work with Professor Hsiao, Chin-Yu.