TPPmark10

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In this problem we consider games like tic-tac-toe or gomoku-narabe, but for simplicity we have only one dimension (all points are aligned).

1. Linear tic-tac-toe

In this game, we play on the integer line $\mathbf{Z}$. Two players, an attacker and a defender, take positions (integers) in turn. A position can be taken only once, and by one player. The attacker plays first. The attacker wins if she can take 3 consecutive positions (*i.e.* x, $x+1$, and $x+2$). The defender succeeds if she has a strategy such that the attacker can never win.

a. Prove that the defender has a succesful strategy.

2. Arithmetic tic-tac-toe

In this game, we play on the integer line $\mathbf{Z}$. Two players, an attacker and a defender, take positions (integers) in turn. A position can be taken only once, and by one player. The attacker plays first. The attacker wins if she can take n equidistant positions (*i.e.* x, $x+d$, $x+2d$, $\ldots x+(n-1)d$ for some $d > 0$). The defender succeeds if she has a strategy such that the attacker can never win.

a. Prove that for $n = 3$ and $n = 4$, an attacker can win against any defender.

b. Prove it also for $n = 5$ (we conjecture this is true).

c. For $n > 6$, try to provide a proof of whether the attacker or the defender have a successful strategy.

Note For arithmetic tic-tac-toe, you may use the rational line $\mathbf{Q}$ instead of $\mathbf{Z}$. They are equivalent for finite games.