# Answers to 「計算と論理」 Exercise Questions Rev. 1.1

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*<2015-08-16 Sun>*

## 1 Basic Information

This file presents Haochen Xie's answers to exercise questions in

`Prof. Jacques Garrigue` による名古屋大学2015年度公開講座・数学アゴラ「計算と論理」

Web link

## 2 Exercise 1.1

### 2.1 Question

Define a Turing machine $M_1$ that converts "unary number" $x$ to binary number $y$.

### 2.2 Input Encoding Definition

A "unary number" has a value equals to the number of 1's it holds. For examples,

$$111_1 = 3_{10}$$
$$111,111,111_1 = 9_{10}$$

The initial state of tape should be like:

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| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\cdots$ | $B$ | $D$ | $x_0$ | $x_1$ | $\cdots$ | $x_{n-1}$ | $B$ | $\cdots$ |

where $x_i = 1$ and $n$ equals to the input number $x$; and the head should be pointing at $D$.

## 2.3 Output Encoding Definition

The final state of tape should be like:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\cdots$ | $B$ | $y_m$ | $\cdots$ | $y_1$ | $y_0$ | $D$ | $E$ | $E$ | $\cdots$ | $E$ | $B$ | $\cdots$ |

where $y = \sum_{i=0}^{m} y_i \cdot 2^i$.

## 2.4 Machine Definition

The machine $M_1$ is defined as

$K = \{RD, A, RT, HT\}$

*read, add, return, halt*

$\Sigma = \{B, 0, 1, D, E\}$

$B$ as in Blank; 0, 1 as number; $D$ as in Divider; $E$ as in Erased

$H = \{HT\}$

$q_0 = RD$

$\delta =$

| $q \backslash a$ | $B$ | $D$ | $E$ | 0 | 1 |
|---|---|---|---|---|---|
| $RD$ | $(B, \rightarrow, HT)$ | $(D, \rightarrow, RD)$ | $(E, \rightarrow, RD)$ | | $(E, \leftarrow, A)$ |
| $A$ | $(1, \rightarrow, RT)$ | $(D, \leftarrow, A)$ | $(E, \leftarrow, A)$ | $(1, \rightarrow, RT)$ | $(0, \leftarrow, A)$ |
| $RT$ | | $(D, \rightarrow, RD)$ | | $(0, \rightarrow, RT)$ | $(1, \rightarrow, RT)$ |

$M_1 = (K, \Sigma, \delta, q_0, H)$

# 3 Exercise 1.2

## 3.1 Question

Define a Turing machine $M_2$ that copy the tape until the symbol $M$ to the end of the tape

## 3.2 The Tape

The initial tape should be like:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\cdots$ | $B$ | $a_k$ | $\cdots$ | $a_0$ | $M$ | $b_i$ | $\cdots$ | $b_0$ | $B$ | $\cdots$ |

where the head should be pointing at $M$.

## 3.3 Machine Definition

The machine $M_2$ is defined as

$K = \{RT, RD, C_0, C_1, RST, HT\}$

*return, read, copy 0/1, restore, halt*

$\Sigma = \{B, 0, 1, M, P_0, P_1\}$

$B$ as in Blank; 0, 1 as number; $M$ as a separater; $P_{0/1}$ as in Processed 0/1

$H = \{HT\}$

$q_0 = RT$

$\delta =$

| $q\backslash a$ | $B$ | $0$ | $1$ | $M$ | $P_y$ |
|---|---|---|---|---|---|
| $RT$ | $(B, \rightarrow, RD)$ | $(0, \leftarrow, RT)$ | $(1, \leftarrow, RT)$ | $(M, \leftarrow, RT)$ | $(P_y, \rightarrow, RD)$ |
| $RD$ | | $(P_0, \rightarrow, C_0)$ | $(P_1, \rightarrow, C_1)$ | $(M, \leftarrow, RST)$ | |
| $C_x$ | $(x, \leftarrow, RT)$ | $(0, \rightarrow, C_x)$ | $(1, \rightarrow, C_x)$ | $(M, \rightarrow, C_x)$ | |
| $RST$ | $(B, \rightarrow, HT)$ | | | | $(y, \leftarrow, RST)$ |

$M_2 = (K, \Sigma, \delta, q_0, H)$

## 4 Exercise 2

**Theorem.** 任意の機械が空テープに適用されたときに停止するかどうかを判定する具体的な手続きは存在しない。

*Proof.* 任意の機械 $M = (K, \Sigma, \delta, q_0, H)$ 及び $M$ に適用できるテープ $T$ に対し，次のように $M' = (K', \Sigma, \delta', {q_0}', H)$ を構築する，但し $RSTT_k \notin K$ $(k \in \mathbb{Z})$ とする。

$$\begin{aligned} a &= min\{i \mid T(i) \neq B\} \\ b &= max\{i \mid T(i) \neq B\} \end{aligned}$$

$$\begin{aligned} K' &= K \cup \{RSTT_i \mid a \leq i \leq b\} \\ {q_0}' &= RSTT_a \\ \delta' &= \delta \cup \{(RSTT_i, B) \mapsto (T(i), \rightarrow, RSTT_{i+1}) \mid a \leq i < b\} \\ &\quad \cup \{(RSTT_b, B) \mapsto (T(b), \rightarrow, q_0)\} \end{aligned}$$

そうすると，新しい機械 $M'$ の空テープ停止問題は機械 $M$ のテープ $T$ に対する停止問題と一致する。 □