Jacques Garrigue, 2012/7/28

Surveys in Mathematical Sciences I (Summer 2012)
Report for Part 3

Errata: Updated parts in problem B are in red.

Report delivery and deadline

You should deliver your report to the support office (支援室) by Monday 2012/8/6.

You can write your report either in English or Japanese, but English is prefered.

Task

You should solve **both** problems A and B.

Problem A

A λ-term is in normal form if the β-rule cannot be applied anywhere inside it.

1. Compute the normal form of $(\mathsf{c}_2\ (\mathsf{c}_2 f)\ (f\ x))$.
2. Compute the normal form of $(\mathsf{c}_3\ (\lambda p.\lambda f.p\ (\lambda x.\lambda y.f\ y\ x))\ (\lambda f.f\ a\ b))$
3. Write a λ-term $\mathsf{c}_{\log}$ computing the base 2 logarithm of its argument m, or more precisely the smallest positive integer n such that $2^n \geq m$.

   Hint: you shall only need to use c_+, $\mathsf{c}_\times$, c_- and $\mathsf{if0}$ to do that.

Erratum: the definition of s' in the lecture notes is wrong. The right definition is:

$$\mathsf{s}' = \lambda x.(\mathsf{pair}\ (\mathsf{snd}\ x)\ (\mathsf{s}\ (\mathsf{snd}\ x)))$$

Problem B

Write the typing derivation for the following term, using the typing rules of the simply typed λ-calculus.

$$\mathsf{Y}_{(\sigma\to\sigma)\to\sigma}\ (\lambda f{:}\sigma.\lambda m{:}\mathrm{int}.\lambda n{:}\mathrm{int}.\mathsf{if0}_{\mathrm{int}\to\sigma}\ m\ n\ (f\ (\mathsf{mod}_\sigma\ n\ m)\ m))$$

where $\sigma = \mathrm{int} \to (\mathrm{int} \to \mathrm{int})$ and $\mathsf{mod}_\sigma\ n\ m$ is the remainder of the division of n by m.

What does this function compute?