# Surveys in Mathematical Sciences I (Summer 2009) Report for Part 2

## Report delivery and deadline

You should deliver your report to the support office (      ) by Tuesday 2009/7/14.

You can write your report either in English or Japanese, but English is prefered.

## Task

You should solve **both** problems A and B.

## Problem A

A $\lambda$-term is in normal form if the $\beta$-rule cannot be applied anywhere inside it.

1. Compute the normal form of $(c_1\ f\ x)$.

2. Compute the normal form of $(c_2\ c_+\ c_1\ c_1\ c_1)$.

3. Write a $\lambda$-term $c_{\sqrt{\ }}$ computing the square root of its argument $m$, or more precisely the smallest positive integer $n$ such that $n \times n \geq m$.

   Hint: you shall only need to use $c_+$, $c_-$ and $if0$ to do that.

## Problem B

Write the typing derivation for the following term, using the typing rules of the simply typed $\lambda$-calculus.

$$Y_{(\sigma \to \sigma)}\ (\lambda f{:}\sigma.\lambda m{:}\mathrm{int}.\lambda n{:}\mathrm{int}.if0_{\mathrm{int} \to \sigma}\ m\ n\ (\mathsf{mod}_\sigma\ n\ m))$$

where $\sigma = \mathrm{int} \to (\mathrm{int} \to \mathrm{int})$ and $\mathsf{mod}_\sigma$ is the quotient remainder.

What does this function compute?