問題 4.2.2〜4.2.7 の解答

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問題 4.2.2

接平面の方程式は

$$z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

法線は接点を通る平面内の各ベクトルと直角な線ので，

$$\begin{cases} (x-a, y-b, z-f(a,b)) \cdot (1, 0, f_x(a,b)) = 0 \\ (x-a, y-b, z-f(a,b)) \cdot (0, 1, f_y(a,b)) = 0 \end{cases} \Leftrightarrow \begin{cases} x - a + (z - f(a,b))f_x(a,b) = 0 \\ y - b + (z - f(a,b))f_y(a,b) = 0 \end{cases}$$

結果的には，$z - f(a,b) = -\dfrac{x-a}{f_x(a,b)} = -\dfrac{y-b}{f_y(a,b)}$

問題 4.2.3

(1) $\dfrac{\partial z}{\partial x} = 6xy + y \quad \dfrac{\partial z}{\partial y} = 3x^2 + x \quad f(1,-1) = -4 \quad f_x(1,-1) = -7 \quad f_y(1,-1) = 4$

接平面 $z + 4 = -7(x-1) + 4(y+1)$ 法線 $z + 4 = \dfrac{x-1}{7} = -\dfrac{y+1}{4}$

(2) $\dfrac{\partial z}{\partial x} = \dfrac{x}{2} \quad \dfrac{\partial z}{\partial y} = \dfrac{2y}{9}$

(3) $\dfrac{\partial z}{\partial x} = \dfrac{y}{(x+y)^2} \quad \dfrac{\partial z}{\partial y} = \dfrac{x}{(x+y)^2}$

(4) $\dfrac{\partial z}{\partial x} = -\dfrac{y}{x^2+y^2} \quad \dfrac{\partial z}{\partial y} = \dfrac{1}{x + y^2/x}$

問題 4.2.7

(1) $$\begin{cases} x = \dfrac{u+v}{2} \\ y = \dfrac{u-v}{2} \end{cases} \Rightarrow \begin{cases} \dfrac{\partial z}{\partial u} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial u} = \dfrac{1}{2}(f_x(\dfrac{u+v}{2}, \dfrac{u-v}{2}) + f_y(\dfrac{u+v}{2}, \dfrac{u-v}{2})) \\ \dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial v} = \dfrac{1}{2}(f_x(\dfrac{u+v}{2}, \dfrac{u-v}{2}) - f_y(\dfrac{u+v}{2}, \dfrac{u-v}{2})) \end{cases}$$

(2) $f(x,y) = g(x+y)$ と仮定すると，$f_x(x,y) = \dfrac{\partial x+y}{\partial x} g'(x+y) = g'(x+y)$ および $f_y(x,y) = \dfrac{\partial x+y}{\partial y} g'(x+y) = g'(x+y)$ から， $f_x(x+y) = f_y(x+y)$ は必要条件である．

(1) の座標変換を使うと，$f(x,y) = f(\dfrac{u+v}{2}, \dfrac{u-v}{2})$ とおける．$f_x(x,y) = f_y(x,y)$ ならば，$\dfrac{\partial z}{\partial v} = 0$ なので，f は $u = x+y$ のみの関数であり，$f(x,y) = g(x+y)$ と書ける．